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III

PROBLEMS IN
HEAT, LIGHT, AND SOUND.

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BY

R. WALLACE STEWART, D.Sc. Lond.,
AUTHOR OF "A HIGHER TEXT-BOOK OF HEAT,"
"A HIGHER TEXT-BOOK OF MAGNETISM AND ELECTRICITY," ETC.,

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PREFACE.

THIS book is issued mainly as a supplement to *Matriculation Physics: Heat, Light, and Sound*, and is not therefore intended to be complete in itself. It is the general experience of Science-teachers that students require a fuller explanation of the mathematical difficulties involved in this subject and far more examples for practice than it is advisable to include within the limits of an ordinary text-book, and it is hoped that this book will be found to provide the necessary mathematical training. Some of the sections and examples are, however, above the standard required for London Matriculation, and these are indicated by an asterisk.

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CHAPTER I.

THERMOMETRY.

13. It is often necessary to convert temperatures expressed in one scale into the corresponding temperatures on either of the other scales. In doing so there are two things to be noticed: (1) Since the interval of temperature between the freezing and boiling points is constant, it follows that 180 Fahrenheit degrees = 100 Centigrade degrees = 80 Réaumur degrees. (2) The zero of the Fahrenheit scale is 32 degrees below freezing point—*i.e.*, 40° F. indicates a temperature 8 degrees F. above freezing point. If, therefore, F, C, and R denote corresponding readings on the Fahrenheit, Centigrade, and Réaumur scales respectively, we have that (F – 32), C and R denote, in each case, the number of degrees the given temperature is above freezing point. Hence, from (1) above—

$$(F - 32) : C : R :: 180 : 100 : 80.$$

That is—

$$(F - 32) : C : R :: 9 : 5 : 4.$$

This proportion may be written thus—

$$\frac{(F - 32)}{9} = \frac{C}{5} = \frac{R}{4}.$$

This should be remembered, and applied, in all cases where it is required to convert temperatures from one scale into another.

Examples I.

1. Find the temperature, on the Fahrenheit scale, corresponding to 40° C.

Here the two scales involved are Fahrenheit and Centigrade. Hence, we write—

$$\frac{F - 32}{9} = \frac{C}{5}.$$

Substituting 40 for C, we have—

$$\frac{F - 32}{9} = \frac{40}{5} = 8.$$

Or—

$$F - 32 = 72.$$

Therefore—

$$F = 104.$$

That is, 40° C. corresponds to 104° F.

2. Find the temperature, on Réaumur's scale, corresponding to $-40°$ F.

As above—

$$\frac{F-32}{9}=\frac{R}{4}.$$

Here—

$$\frac{-40-32}{9}=\frac{R}{4}.$$

Or—

$$\frac{-72}{9}=\frac{R}{4}.$$

That is—

$$\frac{R}{4}=-8.$$

Or—

$$R=-32.$$

3. Find what temperature, on Fahrenheit's scale, is represented by the same number on the Centigrade scale.

Again—

$$\frac{F-32}{9}=\frac{C}{5}.$$

Let T denote the required temperature.

Then—

$$\frac{T-32}{9}=\frac{T}{5}.$$

Or—

$$5T-160=9T$$

$$4T=-160. \quad \therefore T=-40.$$

Hence $-40°$ C. corresponds to $-40°$ F.

4. Find the temperatures on each of the two other scales corresponding to—

(1) $-70°$ C. (2) 76° F. (3) $-24°$ R. (4) 0° C. (5) 50° F.
(6) 68° F. (7) 64° R. (8) 92° C. (9) 14° F. (10) 50° R.

14. Before reading the following chapters on expansion, it will be well for the student to thoroughly master the following points :—

1. The square, or cube (or higher power) of any *small* quantity, or the product of two small quantities, is negligibly small.

Thus—

$$(\cdot 0002)^2=\cdot 00000004.$$
$$(\cdot 0002)^3=\cdot 000000000008.$$

Also—

$${\cdot}0002 \times {\cdot}0003 = {\cdot}00000006.$$

It is evident that if the quantities ·0002 and ·0003 are *small* compared with any quantity, then these three products are *negligibly small* compared with the *same* quantity.

2. Suppose α and β to be small quantities, *compared with unity*, and consider the following relations—

$$(1 + \alpha)^2 = 1 + 2\alpha + \alpha^2.$$

Now, by (1), α^2 is negligible; hence—

$$(1 + \alpha)^2 = 1 + 2\alpha ;$$

and, similarly—

$$(1 + \alpha)^3 = 1 + 3\alpha + 3\alpha^2 + \alpha^3 = 1 + 3\alpha.$$

Also—

$$(1 + \alpha)(1 + \beta) = 1 + \alpha + \beta + \alpha\beta = 1 + \alpha + \beta$$

($\alpha\beta$ being negligible as the product of two small quantities)

$$\frac{1}{1+\alpha} = 1 - \alpha + \alpha^2 - \alpha^3 +, \text{ etc., by actual division, } i.e., \frac{1}{1+\alpha} = 1 - \alpha.$$

$$\frac{1+\alpha}{1+\beta} = 1 + \alpha - \beta - \alpha\beta +, \text{ etc.}, = 1 + \alpha - \beta.$$

It may be useful to tabulate these *approximate* results for future use and reference.

$$(1 + \alpha)(1 + \beta) = 1 + \alpha + \beta.$$

Or generally—

$$(1 \pm \alpha)(1 \pm \beta) = 1 \pm \alpha \pm \beta.$$

$$(1 \pm \alpha)^2 = 1 \pm 2\alpha.$$

$$(1 \pm \alpha)^3 = 1 \pm 3\alpha.$$

$$\frac{1}{1+\alpha} = 1 - \alpha ; \quad \frac{1}{1-\alpha} = 1 + \alpha.$$

$$\frac{1+\alpha}{1+\beta} = 1 + \alpha - \beta.$$

EXAMPLES II.

Find the approximate value of—

(1) $(1{\cdot}000024)(1{\cdot}000065)$. $[(1 + \alpha)(1 + \beta)]$.

(2) $(1{\cdot}00018)({\cdot}99982)$. $[(1 + \alpha)(1 - \beta)]$.

(3) $(1{\cdot}00035)^2 ; ({\cdot}999987)^3$. $[(1 + \alpha)^2, (1 - \alpha)^3]$.

(4) $\dfrac{1}{1{\cdot}000025}; \quad \dfrac{1}{0{\cdot}9987}.$ $\left[\left(\dfrac{1}{1+\alpha}\right), \left(\dfrac{1}{1-\alpha}\right)\right].$

(5) $\dfrac{1{\cdot}00016}{1{\cdot}00004}; \quad \dfrac{1{\cdot}00018}{0{\cdot}99986}$ $\left[\dfrac{1+\alpha}{1+\beta}; \quad \dfrac{1+\alpha}{1-\beta}\right].$

(6) $100\left[\dfrac{1 + 10({\cdot}000064)}{1 - 25({\cdot}000008)}\right]; \quad \dfrac{100}{1{\cdot}00016}.$ $\left[\dfrac{n}{1+\alpha} = n\dfrac{1}{1+\alpha}\right].$

*3. Limiting value of a ratio. Consider the ratio $\frac{a}{x}$. Now, if a is a constant quantity, the fraction $\frac{a}{x}$ can be made as small as we please by sufficiently increasing x. (Thus, if $a = 6$, then, for $x = 1$, $\frac{a}{x} = \frac{6}{1} = 6$; for $x = 1000$, $\frac{a}{x} = \frac{6}{1000} = \cdot 006$; and so on.) Hence, if x be made large enough, we can reduce the value of $\frac{a}{x}$ to a quantity less than any assignable quantity. This is usually stated by saying that the limiting value (or the limit) of $\frac{a}{x}$ is zero when x is infinitely great. Similarly, the limit of $\frac{a}{x}$, when x is zero, is infinite; for the smaller x is, the greater $\frac{a}{x}$ becomes.

Now consider the ratio $\frac{ax^2 + bx}{x}$. What will be the limit of this ratio when $x = 0$. If we substitute $x = 0$ directly, the fraction reduces to $\frac{0}{0}$, which may mean anything; but the ratio evidently equals $ax + b$, and, when $x = 0$, this becomes b; i.e., b is the limiting value of $\frac{ax^2 + bx}{x}$ when $x = 0$. Thus, *although both numerator and denominator reduce to zero for* $x = 0$, *the ratio has a definite limiting value*, b, *for this value of* x.

*4. If V denote the volume, d the density, and M the mass of a body, we have $Vd = M$. Hence, *so long as the mass remains constant*, we may write $Vd =$ *constant*; *i.e.*, if V change to V' and d to d', we have $Vd = V'd'$.

*5. The volume of a column of *uniform* cross-section is given by the product of the height (h) of the column into the area of its cross-section (s); *i.e.*, $V = hs$.

*6. The hydrostatic pressure, on any surface immersed in a liquid, is equal to the *weight* of a column of liquid, of height equal to the depth of the centre of gravity of the surface, and having a cross-section equal in area to that of the surface. That is, $P = hsdg$ where g is the acceleration due to gravity. The pressure on unit surface is given by $p = hdg$.

*7. To find the pressure in the space A under the conditions indicated in Fig. 13. The end, B, of the tube is open to the air. Consider the pressure on each side of the section ab. On one side we have $\Pi s + Hdsg$ where Π is the pressure of the air on unit area and s

is the area of ab. On the other, we have $Ps + hdsg$ where P is the pressure in A on unit area. Since ab is in equilibrium, we have—

$$\Pi s + Hdsg = Ps + hdsg.$$

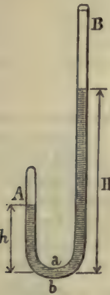

Fig. 13.

We see here that s can be struck out, so that the investigation is independent of s; hence, in similar questions, it will be simpler to consider pressure on *unit area* only. Thus, we have $P + hdg = \Pi + Hdg$ $\therefore P = \Pi + (H - h)dg$, and $P - \Pi = (H - h)dg$. That is, the difference of pressure between the ends A and B is that due to the difference in height of the columns of mercury in the corresponding limbs of the tube.

8. If a quantity, a, varies with b directly, it is expressed thus:—$a \propto b$, or $a = kb$ where k is a constant. For example, the area of a rectangle, of constant breadth, varies with its length; that is, the greater the length, the greater the area. This is expressed by writing $A \propto l$, or $A = kl$, and here the constant k is the breadth of the rectangle.

If a quantity, a, varies inversely with b, we may express this relation in the form $a \propto \frac{1}{b}$, or $a = \frac{k}{b}$; i.e., $ab = k =$ *constant.*

For example, the velocity of a body in travelling over a given space varies inversely with the time it takes; that is, the greater the time, the less the velocity. This is expressed by writing—

$$v \propto \frac{1}{t}, \text{ or } v = \frac{k}{t};$$

and here k is the measure of the given space, for we know from kinematics that $v = \frac{s}{t}$.

CHAPTER II.

EXPANSION OF SOLIDS.

23. WE shall consider calculations involving only the mean coefficients of expansion. The experimental data, for determining true coefficients of expansion, may be obtained by the methods described; but the method of reducing these data cannot be considered here. It should be noticed that, in defining the mean coefficient of expansion, we have to consider the ratio of the mean expansion, per degree rise of temperature, to the original length or volume, as the case may be, **at 0° C.** Thus, if the length of a rod be L_t at $t°$ C., and $L_{t'}$ at a lower temperature $t'°$ C., then the mean expansion per degree is $\frac{L_t - L_{t'}}{(t-t')}$, and the mean *coefficient* of linear expansion (l), between $t°$ and $t'°$, is given by $l = \frac{L_t - L_{t'}}{L_0\,(t-t')}$, where L_0 is the length of the rod at 0° C. In working examples, in addition to the simple relation, $L_t = L_0\,(1 + lt)$, it is useful to have a relation between L_t and $L_{t'}$. This relation is easily obtained thus :—

$$\frac{L_{t'}}{L_t} = \frac{L_0\,(1+lt')}{L_0\,(1+lt)} = \frac{1+lt'}{1+lt}. \qquad \text{(Art. 16.)}$$

Or—

$$L_{t'} = L_t\,\frac{1+lt'}{1+lt}. \qquad (1)$$

Since l is a very small quantity, we may, if t and t' are not very great, consider lt and lt' as small quantities, and apply the approximation of Art. **14**. This gives—

$$L_{t'} = L_t(1 + lt' - lt).$$

That is—

$$L_{t'} = L_t[1 + l(t' - t)]. \qquad (2)$$

This establishes a relation between $L_{t'}$, $L_{t'}$, and l, which is very useful in calculating any one of these quantities, when the other two are given.

Similarly, in addition to $V_t = V_0\,(1+ct)$, we have—

$$V_{t'} = V_t[1 + c(t - t)]. \qquad (3)$$

It is very important to remember that these are only approximate relations, which are nearly true only when l and c are very small. They should therefore be applied only to the expansion of solids.

In the case of liquids, the application of the relation of formula (3) gives a rough approximation, which is sufficiently accurate for most purposes. For accuracy, a formula corresponding to (1) should be employed.

In the case of gases the expansion is far too great to admit of these approximations.

EXAMPLES III.

1. The length of an iron rod at 0° C. is 100 cm. Find its length at 10° C., the mean coefficient of linear expansion of iron being ·000012.

Here—

$$\begin{aligned} L_t &= L_0(1 + lt) \\ \therefore L_t &= 100[1 + (\cdot000012 \times 10)] \\ &= 100[1\cdot00012] \doteq 100\cdot012 \text{ cm.} \end{aligned}$$

2. The volume of a piece of glass at 100° C. is 100·258 c.cm., and its volume at 0° C. is 100 c.c. Find the mean coefficient of cubical expansion of glass between 0° C. and 100° C., and thence deduce approximately the mean coefficient of linear expansion between the same limits of temperature.

$$V_t = V_0(1 + ct).$$

Or—

$$c = \frac{V_t - V_0}{V_0\, t}.$$

Here then—

$$c = \frac{100\cdot258 - 100}{100 \times 100} = \frac{\cdot258}{10000} = \cdot0000258.$$

We have seen that, when l is small, $c = 3l$ or $l = \frac{c}{3}$. Hence, the mean coefficient of linear expansion, as required, is—

$$\frac{\cdot0000258}{3} = 0\cdot0000086.$$

3. The length of a copper rod at 10° is 200·034 cm. Find its length at 100° C., the mean coefficient of linear expansion of copper being ·000017.

Applying formula (2) to data of the question, we have—

$$L_{100} = L_{10}(1 + 90l).$$

That is—

$$\begin{aligned} L_{100} &= 200\cdot034[1 + (90 \times \cdot000017)] \text{ cm.} \\ &= 200\cdot034(1\cdot00153) \text{ cm.} \\ &= 200\cdot34005 \text{ cm. (approximately).} \end{aligned}$$

The accurate length at 100° found by employing formula (1) is 200·34 cm.

4. A brass rod is found to measure 100·019 cm. at 10° C., and 100·19 cm. at 100° C. Find the mean coefficient of linear expansion of brass between 10° C. and 100° C.

Here we may apply formula (2) above—

$$L_{100} = L_{10}(1+90l).$$

That is—

$$l = \frac{L_{100} - L_{10}}{L_{10} \times 90} = \frac{100{\cdot}19 - 100{\cdot}019}{100{\cdot}019 \times 90}$$

$$= \frac{0{\cdot}171}{100{\cdot}019 \times 90} = {\cdot}00001898 \text{ (nearly)}.$$

This is the approximation. If we work the question by application of the accurate formula (1), we shall find that the approximation is a very close one.

$$L_{t'} = L_t \frac{1+lt'}{1+lt}$$

That is—

$$L_{100} = L_{10} \frac{1+100l}{1+10l}$$

$$100{\cdot}19 = 100{\cdot}019 \frac{1+100l}{1+10l}$$

$$100{\cdot}19(1+10l) = 100{\cdot}019(1+100l)$$

$$10001{\cdot}9l - 1001{\cdot}9l = 100{\cdot}19 - 100{\cdot}019,$$

Or—

$$9000l = {\cdot}171$$

$$\therefore\ l = {\cdot}000019.$$

5. A steel scale measures 100·0165 cm. at 15° C.; at what temperature does it measure exactly one metre? The mean coefficient of linear expansion of steel is ·000011.

Employing formula (2), let t denote the required temperature. Then—

$$L_{t'} = L_t\,[1+l(t'-t)]$$

$$100{\cdot}0165 = 100[1+{\cdot}000011(15-t)]$$

$$100{\cdot}0165 = 100+{\cdot}0011(15-t).$$

$$\therefore\ 15-t = \frac{{\cdot}0165}{{\cdot}0011} = 15.$$

$$\therefore\ t = 0.$$

That is, the scale is correct at 0° C.

6. A glass rod is 1 metre long at 0° C.; find its length at −10° C., the mean coefficient of linear expansion of glass being ·0000086.

It should here be noticed that the mean coefficient of expansion is also the mean coefficient of contraction, for if a substance *expands* $\frac{1}{10000}$ of its length at 0° C. for 1° C. *rise* in temperature, it will *con-*

tract $\frac{1}{10000}$ of its length at 0° C. for 1° C. *fall*, in temperature. If, however, we retain the sign of t in the formula—

$$L_t = L_o (1 + lt)$$

it is applicable to all cases.

Thus—

$$\begin{aligned} L_{-10} &= L_o [1 + l(-10)] \\ &= L_o (1 - 10l). \end{aligned}$$

$$\begin{aligned} \text{Here } \therefore L_{-10} &= 1[1 - 10(\cdot0000086)] \\ &= 1[1 - \cdot000086] \\ &= \cdot999914 \text{ metre} \\ &= 99\cdot9914 \text{ cm.} \end{aligned}$$

7. Show that the mean coefficient of superficial expansion for a given substance is approximately equal to twice the mean coefficient of linear expansion for the same substance.

8. A brass and a steel rod are each one metre long at 10° C.; find the difference in their length at 60° C.

9. A platinum wire is found to be 0 013 cm. longer at 60° C. than at 40° C. Find the length of the wire at 0° C. Why can a platinum wire be easily fused into a glass tube?

10. The volume of a mass of lead at 50° C. is 50 c.cm., and at 80° C. its volume is found to be 50·126 c.cm. Show that the mean coefficient of cubical expansion of lead between 50° C. and 80° C. is approximately 0·000084.

11. A silver rod, one inch in diameter at 0° C, just fits into a copper tube at 100° C. At what temperature will it fit into a glass tube of the same diameter as the copper tube at 0° C.?

12. In an experiment by the method of Laplace and Lavoisier the bar was allowed to expand between 0° C. and 60° C. and the following measurements were made: $AB = 100$ cm. at 0° C., $SS' = 2$ cm., $OB = 50$ cm., $SO = 12\cdot5$ metres (see Fig. 14). Find the mean coefficient of linear expansion of the bar.

*13. In an experiment by Roy and Ramsden's method, the bar was allowed to expand between 0° C. and 80° C., and the following measurements were made: $Ra = 101$ cm., $aa' = 0\cdot1455$ cm. (Fig. 16). Find the mean coefficient of linear expansion of the bar. How many turns of the micrometer screw (·5 mm. pitch) would in this case be necessary to bring the cross threads back into coincidence after expansion?

14. A gridiron pendulum is to be made with bars of iron and copper. Give, with a diagram, the dimensions of a suitable arrangement.

15. A rod of zinc is 60 cm. long at 10° C. At what temperature will it be 60·2 cm. long?

16. A steel scale gives correct measurements at 62° F. What length will a rod 1 metre long at 10° C. appear to be when measured with this scale?

17. A glass rod of volume V at 0° C. exactly fills a hollow brass cylinder at 10° C. Find the volume of the space between the rod and the cylinder at 50° C.

18. Two metal rods have the same length, one metre, at 100° C.; but at 80° C. the difference between their lengths is 0·02 cm. If the mean coefficient of linear expansion of the less expansible rod be 0·000015, find the corresponding coefficient for the other rod, and the length of each rod at 0° C.

19. Find the mean coefficient of linear expansion of iron on the Fahrenheit scale, the initial length being referred to zero on the same scale.

20. The volume of a given mass of lead at $-30°$ C. is represented by V. If c denote the mean coefficient of cubical contraction between 0° C. and $-30°$ C., and c' the mean coefficient of cubical expansion between 0° C. and 100° C., find the volume of the lead at 100° C., and express the mean coefficient of cubical expansion of lead between $-30°$ C. and 100° C. in terms of c and c'.

24. Change of Density with Temperature. We have seen that, in general, when a body is heated it expands—that is, its volume increases—and, since the mass of the body remains constant, it must necessarily follow that its density decreases. For, if V_0 denote the volume at 0° C. and V_t the volume at $t°$, also if d_0 denote the density at 0° C. and d_t the density at $t°$ C., then, since the mass remains constant—

$$V_0 \, d_0 = V_t \, d_t.$$

That is—

$$\frac{d_0}{d_t} = \frac{V_t}{V_0}.$$

But we know that $V_t = V_0 \, (1 + ct)$, where c denotes the coefficient of cubical expansion.

Therefore—

$$\frac{d_0}{d_t} = \frac{V_t}{V_0} = \frac{V_0 \, (1 + ct)}{V_0} = 1 + ct,$$

Or—

$$d_t = \frac{d_0}{1 + ct}. \qquad (1)$$

This is true, as it stands, for solids, liquids, and gases; but for solids and some liquids we may have, when t is small enough, an approximate formula, giving—

$$d_t = \frac{d_0}{1+ct} = d_0 \, \frac{1}{1+ct} = d_0 \, (1 - ct). \qquad \text{[Art. 14 (2).]}$$

Also corresponding to formula (3), above, we have—

$$\frac{d_t}{d_{t'}} = \frac{V_{t'}}{V_t} = \frac{V_0 \, (1 + ct')}{V_0 \, (1 + ct)} = \frac{1 + ct'}{1 + ct} = [1 + c(t - t)]. \qquad (2)$$

EXAMPLES IV.

1. The density of a piece of glass at 10° C. is 2·6, and at 60° C. it is 2·5966. Find the mean coefficient of cubical expansion of glass.

Applying—

$$\frac{d_t}{d_{t'}} = [1 + c(t' - t)],$$

$$\frac{2{\cdot}6}{2{\cdot}5966} = 1 + 50c,$$

$$1{\cdot}00131 = 1 + 50c,$$

$$c = \frac{{\cdot}00131}{50} = {\cdot}0000262.$$

2. The density of mercury at 0° C. is 13·596. Find its density at 100° C., the mean coefficient of cubical expansion of mercury between 0° C. and 100° C. being ·000181.

From—

$$d_t = \frac{d_o}{1 + ct}, \text{ we have—}$$

$$d_t = \frac{13{\cdot}596}{1{\cdot}0181} = 13{\cdot}354.$$

From the approximate relation—

$$d_t = d_o(1 - ct),$$

$$d_t = 13{\cdot}596(1 - {\cdot}0181)$$

$$= 13{\cdot}596({\cdot}9819)$$

$$= 13{\cdot}350,$$

which is a fairly close approximation.

3. Find the weight of a cubic centimetre of mercury at 10° C., having given that, one cubic centimetre of mercury weighs, at 0° C., 13·596 grams, and its mean coefficient of expansion is ·000181.

Weight is proportional to mass, and, since $M = Vd$, mass is proportional to density when the volume is constant. Hence, if w_t and w_o denote the weights of a given volume at t° C. and 0° C., we have—

$$\frac{w_t}{w_o} = \frac{d_t}{d_o} = \frac{1}{1 + ct}.$$

$$\therefore \frac{w_t}{w_o} = 1 - ct, \text{ or, } w_t = w_o(1 - ct).$$

$$\text{Here } \therefore w_t = 13{\cdot}596[1 - (10 \times {\cdot}000181)]$$

$$= 13{\cdot}596 - 13{\cdot}596({\cdot}00181)$$

$$= 13{\cdot}571 \text{ grams.}$$

4. Compare the density of lead at 100° C. with its density at $-100°$ C., assuming its coefficient of expansion to remain constant within these limits of temperature.

5. The mean coefficient of expansion of a substance between $-t°$ C. and 0° C. is c, and the mean coefficient of expansion between 0° C. and T° C. is c'. Compare the density of the substance at each of these temperatures with its density at 0° C.

6. Find the mass of a cubic centimetre of silver at 250° C., the density of silver at 0° C. being 10·31 grams per c.c.

7. The density of water at 0° C. is 0·999871, and at 4° C. it is 1. Find the mean coefficient of expansion of water between 4° C. and 0° C.

8. If δ denote the density of a substance at $t°$ C., and δ' its density at $t'°$ C., find the mean coefficient of expansion of the substance between $t°$ C. and $t'°$ C.

9. Find the weight, in grams, of 10 c.cm. of gold (density 19·4 grams per c.cm. at 0° C.) at 15° C.

10. The density at 0° C. of a specimen of wrought iron is 7·3, and the density at 0° C. of a specimen of tin is 7·4 : at what temperature will these two specimens have the same density ?

CHAPTER III.

EXPANSION OF LIQUIDS.

30. It will here be convenient to collect together the various formulæ deduced in connection with this subject.

1. Apparent and real expansion.

(*a*) $l_r = l_a + l$. Art. 25 (5′).

(*b*) $c_r = c_a + c$. Art. 25 (5).

These two formulæ [(*a*) and (*b*)] should be learnt in words.

(*c*) $L_a = L_0 (1 + l_a t)$. Art. 25 (3′).

(*d*) $V_a = V_0 (1 + c_a t)$. Art. 25 (3).

These formulæ establish a relation between *apparent* length or volume *at t° C.* and *true* length or volume *at 0° C.*

(*e*) $L_t = L_a (1 + lt)$. Art. 25 (1′).

(*f*) $V_t = V_a (1 + ct)$. Art. 25 (1).

These formulæ establish a relation between the *true* length or volume *at t° C.* and the *apparent* length or volume also *at t° C.*

2. Weight thermometer.

(*g*) $c_a = \dfrac{w'}{(W_0 - w')T}$. Art. 27 (1).

(*h*) $T = \dfrac{w'}{(W_0 - w')c_a}$. Art. 27 (2)

*3. Hydrostatic method of determining cubical expansion of a solid or liquid (Art. 28).

(*i*) $w_t (1 + ct) = w_0 (1 + gt)$.

For calculation, this formula may be more conveniently expressed as an approximation, thus :—

$$\frac{w_0}{w_t} = \frac{1 + ct}{1 + gt}.$$

$$\therefore \quad \frac{w_0}{w_t} = 1 + (c - g)t. \qquad (k)$$

Also, as in Art 23, we may have the approximate relation—

$$(l) \qquad \frac{u'_{t'}}{u'_{t}} = 1 + (c - g)(t - t').$$

The reader should make himself thoroughly familiar with these formulæ before going any farther. In studying any formula the following plan should be adopted: (1) Thoroughly master the method by which the relation is obtained. (2) Note carefully the conditions under which it is applicable. (3) If the formula is fundamental, it should be learnt; but, in all cases, the method of work is of more importance than the formulated result.

The following examples illustrate the application of the above relations.

Examples V.

1. A zinc rod is measured by means of a brass scale, and found to be 1·0001 metres long at 10° C. What is the real length of the rod at 0° C. and at t° C.? [Mean coefficient of linear expansion of zinc is ·000029 and of brass ·000019.]

Applying (c) above we get—

$$L_a = L_o(1 + l_a t).$$

$$\therefore L_o = \frac{L_a}{1 + l_a t} = L_a(1 - l_a t).$$

Here—

$$L_a = 1{\cdot}0001 \,;\ l_a = (l_r - l) = ({\cdot}000029 - {\cdot}000019) = {\cdot}00001 \,;\ \text{and } t = 10.$$

$$\begin{aligned} \therefore L_o &= 1{\cdot}0001[1 - ({\cdot}00001)10] \\ &= 1{\cdot}0001[1 - {\cdot}0001] \\ &= 1 \text{ nearly.} \end{aligned}$$

Also—

$$L_t = L_o(1 + l_r t). \qquad \text{(Art. 16.)}$$

Here—

$$L_o = 1 \text{ nearly, and } l_r = 0{\cdot}000029.$$

$$\begin{aligned} \therefore L_{10} &= 1(1 + 0{\cdot}00029) \\ &= 1{\cdot}00029. \end{aligned}$$

Or—

$$\begin{aligned} L_t &= L_a(1 + lt). \\ \therefore L_{10} &= 1{\cdot}0001(1 + {\cdot}000019 \times 10) \\ &= 1{\cdot}0001(1 + {\cdot}00019) \\ &= 1{\cdot}00029\ldots \end{aligned}$$

2. A glass tube is measured with a steel scale, correct at 0° C., and found to be 70 cm. long at 0° C. and 69·99811 cm. long at 10° C.

Find the mean coefficient of real expansion of glass, and also its mean coefficient of apparent expansion with reference to steel. [The mean coefficient of expansion of steel is 0·0000113.]

$$L_o = L_a [1 - l_a t]$$
$$70 = 69{\cdot}99811[1 - 10l_a]$$
$$70 = 69{\cdot}99811 - 699{\cdot}9811 l_a$$
$$699{\cdot}9811 l_a = 69{\cdot}99811 - 70$$
$$699{\cdot}9811 l_a = 0{\cdot}00189.$$
$$\therefore l_a = -\frac{{\cdot}00189}{699{\cdot}9811} = -0{\cdot}0000027.$$

That is, the mean coefficient of apparent expansion of glass relative to steel is negative. This explains why the tube is *apparently* shorter at 10° C. than at 0° C.

Also— $l_r = l + l_a$, and $l = 0{\cdot}0000113$.

$$\therefore l_r = 0{\cdot}0000113 - 0{\cdot}0000027$$
$$= 0{\cdot}0000086.$$

3. Barometric correction for temperature. The pressure of the atmosphere is usually expressed in terms of the height of a column of mercury at 0° C. which exerts an equivalent pressure. The *observed* height of a barometer at $t°$ C. has to be reduced to the *equivalent* height at 0° C.; this is called correcting for temperature, and in applying the correction two points are to be remembered. (i) To correct for the expansion of the scale between 0° C. and $t°$ C. (ii) To correct for the change of density of the mercury. Let H denote the *observed* height of the barometer at $t°$ C. Then if l denote the mean coefficient of linear expansion of the scale, the true height, corrected for the expansion of the scale, is $H(1 + lt)$. Denote this by H_t. Then for (ii) we have, as in Art. **26**, $H_t\, d_t = H_o\, d_o$.

That is— $$H(1 + lt)d_t = H_o\, d_o.$$

$$\therefore H_o = H(1 + lt)\frac{d_t}{d_o}.$$

But— $$\frac{d_t}{d_o} = \frac{1}{1 + c_r t}. \qquad \text{(Art. 24.)}$$

$$\therefore H_o = H\,\frac{1 + lt}{1 + c_r t} = H[1 + (l - c_r)t].$$

Or, since c_r is usually greater than l, this is more generally written

$$H_o = H[1 - (c_r - l)t]$$

where H denotes the observed height at $t°$ C.
H_o „ „ true „ at 0° C.
l „ „ mean coefficient of expansion of the scale.
c_r „ „ „ „ absolute cubical expansion of mercury.
and t „ „ temperature of observation. It should be noticed that the mean coefficient of *absolute cubical* expansion of

mercury is employed because the correction (ii) is necessary on account of change of density of the mercury. On no account must the column of mercury be treated as a rod subject to linear expansion.

A barometer with a glass scale reads 755 mm. at 18° C.; find the reading at 0° C. The apparent coefficient of expansion of mercury in glass is ·000155, and the mean coefficient of linear expansion of glass is ·0000089.

Applying the above formula, we have $H_o = H[1 - (c_r - l)t]$.

To find c_r from c_a which is given, we have from (*b*) $c_r = c_a + c$.

Here $c_a = \cdot 000155$ and $c = 3(\cdot 0000089) = \cdot 0000267$ (Art. 14).

$$\therefore c_r = \cdot 000155 + \cdot 0000267 = \cdot 0001817.$$

Then—

$$\begin{aligned} H_o &= 755[1 - (\cdot 0001817 - \cdot 0000089)18] \\ &= 755[1 - (\cdot 0001728)\,18] \\ &= 755[1 - \cdot 00311] \\ &= 752\cdot 652 \text{ nearly.} \end{aligned}$$

4. An ordinary mercurial thermometer is placed with its bulb and lower part of stem in a vessel of water, and indicates a temperature T. The upper portion of the stem, containing *n* divisions of the mercury column, is in the air at a temperature *t*. What is the true temperature of the water in the vessel?

The true temperature of the water, T′, is that which the thermometer would indicate if completely immersed in the water. If this were the case, the *n* divisions of the mercury column, which are now at t°, would be at T'°. We have therefore to find the expansion of these *n* divisions for a rise in temperature from t° to T'°. If c_a denote the mean coefficient of apparent expansion of mercury in glass we have—

$$n' = n[1 + c_a(T' - t)]$$

But—

$$\begin{aligned} T' &= (T - n) + n' \\ &= T + n' - n \\ &= T + n(T' - t)c_a. \qquad \text{(Cp. Art 12, 4.)} \end{aligned}$$

5. A weight thermometer weighs 50 grams when empty, and 710 grams when full of mercury at 0° C. On heating up to 100° C., 10 grams of mercury are expelled. Calculate the mean coefficient of cubical expansion of glass, assuming the mean coefficient of real cubical expansion of mercury to be 0·000181.

Applying (*g*) above we get—

$$c_a = \frac{w'}{(W_o - w')T}.$$

$$\therefore c_a = \frac{10}{(660-10)100} = \frac{1}{6500}.$$

$$\therefore c_a = 0\cdot 000154 \text{ nearly.}$$

But—

$$c_r = c_a + c. \qquad \textbf{(b)}$$

$$\begin{aligned} \therefore c &= c_r - c_a \\ &= 0\cdot 000181 - 0\cdot 000154 \\ &= 0\cdot 000027. \end{aligned}$$

6. The same weight thermometer is employed to determine the expansion of glycerine. Its weight, full of glycerine at 0° C = 163·13 grams, and at 100° C = 157·65 grams. Find the mean coefficient of real cubical expansion of glycerine.

As above—

$$c_a = \frac{w'}{(W_o - w')T}.$$

Here $w = 5·48$, $W_0 = 163·13 - 50 = 113·13$, and $T = 100$.

$$\therefore\ c_a = \frac{5·48}{(107·65)100} = 0·00051 \text{ nearly.}$$

$$\text{and } c_r = c_a + c$$

$$\therefore\ c_r = 0·00051 + 0·000027 = 0·000537.$$

Examples 5, 6, should be studied in connection with Art. **27** (1).

7. The weight thermometer, of questions 4 and 5, is employed to determine the temperature of an oil bath. After it has taken the temperature of the bath, it is found that 30 grams of mercury have been expelled. Find the temperature of the bath.

From (*h*) we have—

$$T = \frac{w'}{(W_o - w')c_a}.$$

That is—

$$T = \frac{30}{(660-30)\frac{1}{6500}} = \frac{6500 \times 30}{630}.$$

$$\therefore\ T = 309·5° \text{ C. nearly.}$$

8. A glass bulb, with a uniform fine stem, weighs 10 grams when empty, 117·3 grams when the bulb only is full of mercury, and 119·7 grams when a length of 10·4 cm. of the stem is also filled with mercury; calculate the relative coefficient of expansion for temperature of a liquid which, when placed in the same bulb, expands through the length from 10·4 to 12·9 of the stem, when warmed from 0° C. to 28° C. The density of mercury is 13·6 grams per c.cm.

From question, 10·4 cm. of the bore of the stem contains 119·7 − 117·3 = 2·4 grams of mercury. ∴ 1 cm. of bore of stem contains $\frac{2·4}{10·4} = \frac{3}{13}$ grams of mercury.

Let v denote the volume of 1 cm. of bore of stem, then volume of bulb is equivalent to that of $\frac{117·3 - 10}{\frac{3}{13}} v = \frac{107·3 \times 13}{3} v$ cm. of stem $= 464·97v$ nearly.

∴ Initial volume of liquid $= (464·96 + 10·4)v = 475·37v$ nearly, and final volume $= (464·97 + 12·9)v = 477·87v$.

Now, applying formula of Art. **28**—

$$c_a = \frac{n - n}{nT},$$

we get—

$$c_a = \frac{2·5v}{28 \times 475·37v} = \frac{2·5}{475·37 \times 28} = 0·0001878 \text{ nearly.}$$

This is an example of the method described in Art. **28** (2). The operations with mercury, referred to in the first part of question, constitute a simple method of calibration.

A more usual way of working such a question is to deduce the volume of the bulb, etc., in c.cm., by employing the known density of mercury (13·6), thus :—

$$\text{Volume of 1 cm. of stem } \frac{2{\cdot}4}{10{\cdot}4 \times 13{\cdot}6} \text{ c.cm.} = \frac{3}{13 \times 13{\cdot}6} \text{ c.cm.}$$

$$\text{Total apparent expansion} = \frac{2{\cdot}5 \times 3}{13 \times 13{\cdot}6}.$$

∴ Mean coefficient of apparent expansion between 0° C. and 28° C.

$$= \left(\frac{2{\cdot}5 \times 3}{13 \times 13{\cdot}6 \times 28}\right) \div \frac{109{\cdot}7}{13{\cdot}6}$$

$$= \frac{2{\cdot}5 \times 3 \times 13{\cdot}6}{13 \times 13{\cdot}6 \times 28 \times 109{\cdot}7} = \frac{2{\cdot}5 \times 3}{13 \times 28 \times 109{\cdot}7} = 0{\cdot}0001878 \text{ nearly.}$$

*9. A piece of glass weighs 47 grams in air, 31·53 grams in water at 4° C. and 31·75 grams in water at 60° C. Find the mean coefficient of cubical expansion of water between 4° and 60°, taking that of glass as 0·000024.

From (l) we have—

$$\frac{w_4}{w_{60}} = 1 + (c - g)(60 - 4).$$

Here—

$$w_{60} = 47 - 31{\cdot}75 = 15{\cdot}25 \text{ grams,}$$
$$w_4 = 47 - 31{\cdot}53 = 15{\cdot}47 \text{ grams,}$$
$$g = 0{\cdot}000024.$$

Hence—

$$\frac{15{\cdot}47}{15{\cdot}25} = 1 + (c - g)56,$$

$$1{\cdot}014426 = 1 + (c - g)56.$$

$$\therefore c - g = \frac{0{\cdot}014426}{56} = 0{\cdot}000257.$$

That is—

$$c = 0{\cdot}000257 + g.$$

$$\therefore c = 0{\cdot}000257 + 0{\cdot}000024 = 0{\cdot}000281.$$

A simple example such as this may be, very instructively, worked out from first principles. The loss of weight of the glass in water at 4° C. = 15·47 grams ; ∴ its volume = 15·47 c.cm., for a gram is, by definition, the weight of 1 c.cm. of water at 4° C. Hence, volume of glass at 60° $= 15{\cdot}47[1 + (0{\cdot}000024 \times 56)]$
$= 15{\cdot}4908$ c.cm.

Hence, when weighed in water at 60° C., it displaces this volume of water, and therefore we have, that 15·4908 c.cm. of water at 60° weighs

15·25 grams. This weight of water would occupy 15·25 c.cm. at 4° C., and hence we have—

$$15\cdot4908 = 15\cdot25(1 + 56c).$$

$$\therefore\ c = \frac{0\cdot2408}{15\cdot25 \times 56} = 0\cdot000282.$$

This result agrees very closely with the approximation obtained above.

*10. A solid weighs 29·9 grams in a liquid of density 1·21 at 10° C. It weighs 30·4 grams in the same liquid at 95° C. when the density of the liquid is 1·17. Calculate the coefficient of cubical expansion of the solid, given that its weight in air = 45·6 grams.

From (*l*) we have—

$$\frac{w_{t'}}{w_t} = 1 + (c - g)(t - t').$$

Here—

$$w_{t'} = w_{10} = 45\cdot6 - 29\cdot9 = 15\cdot7$$
$$w_t = w_{95} = 45\cdot6 - 30\cdot4 = 15\cdot2.$$

And therefore—

$$\frac{15\cdot7}{15\cdot2} = 1 + (c - g)(95 - 10),$$

or—

$$1\cdot0329 = 1 + 85(c - g).$$

$$\therefore\ c - g = \frac{\cdot0329}{85} = \cdot000387.$$

Here *g* is required and *c* has to be calculated from the densities of liquid at 10° C. and 95° C. From Art. **24** we have—

$$\frac{d_{10}}{d_{95}} = [1 + (95 - 10)c] = 1 + 85c.$$

That is—

$$\frac{1\cdot21}{1\cdot17} = 1 + 85c,$$

or—

$$1\cdot034188 = 1 + 85c.$$

$$\therefore\ c = \frac{0\cdot034188}{85} = 0\cdot000402.$$

Hence for *g* we have—

$$0\cdot000402 - g = 0\cdot000387.$$
$$\therefore\ g = 0\cdot000015.$$

11. A rod of copper and a rod of iron placed side by side are riveted together at one end. The iron rod is 150 cm. long, and a mark is made on the copper rod, showing the position of the unriveted end of the iron at 0° C. If at 30° the mark is 0·0255 cm. from the end of the iron rod, what is the coefficient of expansion of copper, that of iron being 0·000012?

*12. Deduce from the table on page 67 the mean coefficient of cubical expansion of water between 0° C. and 4° C.; 4° C. and 10° C. and 4° C. and 20° C.

13. In an experiment made by Dulong and Petit's method to determine the absolute expansion of mercury, the following data were obtained: $h_t - h_0 = 0{\cdot}91$ cm., $h_0 = 50$ cm. The temperature of the columns of mercury being respectively 0° C. and 100° C., what value does this experiment give for the coefficient of absolute expansion of mercury?

14. The weight thermometer used in the experiment of question 13 weighed 95·6 grams when empty, and 676·8 grams when full of mercury at 0° C.; find the quantity of mercury expelled during the experiment.

15. In an experiment by Regnault's method for the determination of the absolute expansion of mercury, the following data were obtained: $\mathrm{H} = 100$ cm.; $h = 12{\cdot}4$ cm.; $h' = 15{\cdot}97$ cm.; $t = 10°$ C.; $\mathrm{T} = 210°$ C. Find the value of c given by this experiment.

16. The coefficient of absolute cubical expansion of mercury is ·00018, the coefficient of linear expansion of glass is ·000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101·56 divisions?

17. A porcelain weight thermometer weighs 165 grams when empty and 468 grams when full of mercury at 0° C. When heated to 300° C., the weight of overflow is found to be 13·464 grams. Find the mean coefficient of cubical expansion of porcelain between 0° and 300° C., assuming that of mercury to be ·000184 for the same range of temperature.

*18. A solid weighs 320 grams *in vacuo*, 240 grams in distilled water at 4° C., and 242 grams in water at 100° C., of which the density is 0·959. Find the volume of the solid at these two temperatures, and deduce therefrom its mean coefficient of cubical expansion for 1° C.

19. The barometer height at 12° C., as indicated by a barometer with a brass scale, is 766·45 cm. Find the true equivalent height at 0° C.

20. A glass bulb with a straight graduated capillary stem weighs 54 grams when empty, 367 grams when filled with mercury, at 0° C., up to the zero of the graduations on the stem, and 367·12 grams when filled with mercury, at 0° C., up to the division marked 100 on the stem. Find the mean coefficient of absolute cubical expansion of a liquid which fills the bulb and stem up to the 10th division at 0° C. and up to the 110th division at 100° C.

21. If δ be the expansion of water between 4° and 0° C., and Δ its expansion between 4° and $t°$, show what is the density of water at $t°$ referred to water at 0°.

CHAPTER IV.

EXPANSION OF GASES.

In connection with the expansion of gases the formulated expressions of the gaseous laws of Art. **32** should be noted :—

(1) **Boyle's Law.** $pv = k$ [Art. **32** (i).]
(2) **Charles' Law.** $v_t = v_0\ (1 + c_v\, t)$ [„ „ (ii).]
(3) **Law of pressures.** $p_t = p_0\ (1 + c_p\, t)$ [„ „ (iii).]

[In (2) and (3) experiment shows that $c_v = c_p = \frac{1}{273}$ approximately.]

These formulæ are concisely expressed by the relation—

$$\frac{p_1 v_1}{1 + c_v\, t_1} = \frac{p_2 v_2}{1 + c_v\, t_2},$$

or more conveniently, for the purposes of calculation, by—

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2},$$

which is the general expression of formulæ (1) and (2) of Art. **34**.

The last formula given involves six quantities, and it is evident that, if any five be given, the remaining one can be calculated. If any one of the variables (p, v, T) involved be supposed constant it cancels out of the equation ; thus, at constant temperature, we have—

$$p_1 v_1 = p_2 v_2 \qquad (1\ a)$$

at constant pressure,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \quad \therefore \frac{v_1}{v_2} = \frac{T_1}{T_2} \qquad (1\ b).$$ (Compare Art. **34**.)

and at constant volume,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \therefore \frac{p_1}{p_2} = \frac{T_1}{T_2} \qquad (1\ c).$$ (Compare Art. **34**.)

When the coefficient of expansion (c_v) is given, formula (iv) of Art. **32** should be adapted in a similar way to the conditions of the problem. (See Ex. 5.)

The approximate relations of preceding chapters should not be used.

EXAMPLES VI.

1. A cubic metre of gas at 760 mm. pressure is subjected, at constant temperature, to a pressure of 2280 mm. Find its volume.

Here, in—

$$p_1 v_1 = p_2\, v_2 \qquad (1\ a).$$

we have—

$p_1 = 760$; $p_2 = 2280$ mm.
$v_1 = 1$ cubic metre ; v_2 is required.

Hence—

$$760 \times 1 = 2280\, v_2.$$

Or—

$$v_2 = \frac{760}{2280} = \frac{1}{3} \text{ cubic metre.}$$

2. A litre of hydrogen, at 10° C., is heated at constant pressure to 293° C. Find its volume.

Here, in—

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \qquad (1\ b).$$

we have—

$$v_1 = 1 \text{ litre}; \ T_1 = 273 + 10 = 283,$$
$$T_2 = 273 + 293 = 566; \ v_2 \text{ is required.}$$

Hence—

$$\frac{1}{v_2} = \frac{283}{566} = \frac{1}{2},$$
$$\therefore v_2 = 2 \text{ litres.}$$

3. Air is enclosed in a vessel at 0° C., and, the volume being kept constant, the temperature is lowered to −88° C., at which temperature the pressure is found to be 385 mm. Find the pressure at 0° C.

Here, in—

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

we have—

$$p_1 \text{ is required}; \ T_1 = 273.$$
$$p_2 = 385 \text{ mm.}; \ T_2 = 273 + (-88) = 273 - 88 = 185.$$

Hence—

$$\frac{p_1}{385} = \frac{273}{185},$$
$$\therefore p_1 = 568{\cdot}1 \text{ mm.}$$

4. Find the volume occupied at 0° C. and 760 mm. pressure by 500 c.cm. of oxygen measured at 10° C. and 750 mm.

Here, in—

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

we have—

$$p_1 = 750 \text{ mm.}; \ p_2 = 760 \text{ mm.}$$
$$v_1 = 500 \text{ c.c.}; \ v_2 \text{ is required.}$$
$$T_1 = 273 + 10 = 283; \ T_2 = 273 + 0 = 273.$$

Hence—

$$\frac{750 \times 500}{283} = \frac{760 v_2}{273}.$$

Or—

$$v_2 = \frac{273 \times 750 \times 500}{283 \times 760}$$

$$= 480\cdot 635 \text{ c.cm.}$$

5. A thousand cubic centimetres of air at 50° C. are cooled down to 10° C., and at the same time the external pressure upon the air is increased from 750 mm. to 765 mm. What is the volume reduced to, the coefficient of expansion of air for 1° C. being 0·00366?

Here, applying formula (iv) of Art. 32, viz.—

$$\frac{p_1 v_1}{1 + c_r t_1} = \frac{p_2 v_2}{1 + c_r t_2},$$

we have—

$p_1 = 750$ mm. $p_2 = 765$ mm.
$v_1 = 1000$ c.cm. v_2 is required.
$t_1 = 50°$ C. $t_2 = 10°$ C.
$c_r = 0\cdot 00366.$

$$\therefore \frac{750 \times 1000}{1 + (0\cdot 00366 \times 50)} = \frac{765 v_2}{1 + (0\cdot 00366 \times 10)}.$$

$$\therefore \frac{750 \times 1000}{1\cdot 183} = \frac{765 v^2}{1\cdot 0366}.$$

$$\therefore v_2 = 859\cdot 06 \text{ c.cm.}$$

6. Compare the density of air at 0° C. with that of air at 100° C.

If we take a given mass of air at 0° C., and heat it to 100° C., the density diminishes; but, since the mass is constant, we have, at any temperature $v_1 d_1 = v_2 d_2$;

$$i.e., \quad \frac{v_1}{v_2} = \frac{d_2}{d_1}.$$

But from (1 *b*) we have—

$$\frac{v_1}{v_2} = \frac{T_1}{T_2}.$$

$$\therefore \quad \frac{d_2}{d_1} = \frac{T_1}{T_2}. \qquad (3)$$

[That is, density is inversely proportional to absolute temperature. Compare Art. 24.]

Here then—

$$\frac{d_0}{d_{100}} = \frac{373}{273} = 1\cdot 366$$

7. The mass of a litre of air at 0° C. and 760 mm. pressure is 1·293 grams; find the mass of a litre of air at 100° C. and 750 mm. pressure.

The volume being constant, mass is proportional to density, that is—

$$\frac{M}{M'} = \frac{d}{d'}.$$

But, as in Example 6, $\frac{d}{d'} = \frac{T'}{T}$,

and, by Boyle's law, $\frac{d}{d'} = \frac{p}{p'}$. $\therefore \frac{d}{d'} = \frac{T'}{T} \cdot \frac{p}{p'}$;

$$\text{and } \frac{M}{M'} = \frac{d}{d'} = \frac{T'}{T} \cdot \frac{p}{p'}.$$

$$\therefore M' = M \cdot \frac{T}{T'} \cdot \frac{p'}{p}.$$

Here, $M = 1{\cdot}293$ grams, $T = 273, p = 760$.
M' is required, $T' = 373, p' = 750$.

$$\therefore M' = 1{\cdot}293 \times \frac{273}{373} \times \frac{750}{760}{}^{*} = {\cdot}9339 \text{ grams.}$$

8. In a determination of the coefficient of expansion of air by Gay Lussac's method the volume of the air in the tube was found to be 240 c.cm. at 0° C., and at 77° C. its apparent volume was 310 c.cm. Find the value obtained for the required coefficient. The mean coefficient of cubical expansion of glass is 0·000026.

*9. In an experiment with the apparatus of Fig. 32 it is found that at 0° C. $V_m = 506$ c.cm., $V_{m'} = 686$ c.cm., and $\frac{v}{n} = 1{\cdot}5$ c.cm. The cylinder L L is first filled with ice, and the level of the mercury adjusted to *m*. The ice is then removed, and a rapid current of steam at 100° C. passed through. On final adjustment of the mercury it is found that the level of the column coincides with the third division of the scale *m′ m″*. Find the coefficient of expansion of air, given the coefficient of cubical expansion of glass is ·000026.

*10. The weight thermometer of question 14 (Ex. V.) is used to determine the mean coefficient of expansion of air by the method of Art. 33. Determine w, given $g = {\cdot}000026$ and c_v for air $= 0{\cdot}003665$.

*11. A porcelain air thermometer is used to determine the temperature of a furnace. The excess of the pressure of the air in the bulb over the atmospheric pressure is found to be that due to 1843 mm. of mercury. Find the temperature of the furnace, given that the barometric height at the time of determination equals 758 mm.

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* This, and similar relations, can be written down at once by noticing whether the given changes of temperature and pressure involve, respectively, an increase or decrease of the quantity considered, and then arranging the ratios accordingly.

12. A volume V of gas at pressure P and temperature T is heated (1) at constant pressure and (2) at constant volume to a temperature T′. Express in terms of P, V, T, and T′ the resulting *state* in each case. [The *state* of a given mass of gas is expressed by giving its pressure, volume, and temperature.]

13. Ten litres of hydrogen at 20° C. and 750 mm. pressure occupy a volume of 9532·4 c.cm. at 10° C. and 760 mm. pressure. Find the mean coefficient of expansion of hydrogen.

14. A straight vertical tube, the section of whose bore is one square inch, is closed at its lower end and contains a quantity of air, which supports an air-tight piston whose weight is 1 lb. The position of the piston is observed when the temperature of the air is 31° C., and the weight of the piston is then increased by 1 lb. Find what increase of temperature will be required to bring back the piston to its former position, the atmospheric pressure being 15 lb. per square inch, and the absolute zero of the air thermometer being $-273°$ C.

15. Determine the height of the barometer when a milligramme of air at 27° C. occupies a volume of 20 c.cm. in a tube over mercury, the mercury standing 73 cm. higher inside the tube than outside. (1 gram of air at 0° C. under a pressure of 76 cm. of mercury measures 773·4 c.cm.)

16. The volume of a bubble of gas, generated under water at a depth of 100 metres, is 2 c.cm. Find its volume when it reaches the surface assuming the temperature to increase 1° C. for each 10 metres rise, the temperature at the surface to be 15° C., and the pressure of the air to be equal to that due to 10 metres of water.

17. A litre of hydrogen weighs 0·0896 gram at 0° C. and 760 mm. pressure. Find the weight of a litre at 20° C. and 766 mm. pressure.

18. Compare the density of air at 10° C. and 750 mm. pressure with its density at 15° C. and 760 mm. pressure.

19. A flask containing air is corked up at 20° C. and heated in an air bath. A pressure of two atmospheres inside the flask will force the cork out; at what temperature will this take place?

20. At what temperature will a volume of air at 0° C., when heated at constant pressure, double its volume?

21. A thin spherical glass bulb, 2 cm. in diameter, containing air is sealed up and enclosed in a spherical vessel 10 cm. in diameter, and containing the same quantity of air as the bulb. The temperature in both vessels is then raised until the inner one bursts. The pressure in the enclosing vessel is then found to be 1·5 atmospheres; find the pressure within the bulb just before bursting.

22. Find the temperature of absolute zero on the Fahrenheit scale.

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

Elementary.

1. Describe the process of filling and closing a mercurial thermometer. It is sometimes found that when a mercurial thermometer is placed in melting ice, the temperature indicated is not zero, but a fraction of a degree above zero: how is this explained?

2. Describe an apparatus by which the Coefficient of Expansion of a Gas may be found. 500 cubic centimetres of Oxygen Gas are measured when the temperature is 20° C., and the temperature is then raised to 40° C., the pressure meanwhile remaining invariable. What is the volume of the Oxygen at the latter temperature? (The Coefficient of Expansion of Oxygen is $\frac{11}{3000}$.)

3. A thousand cubic inches of air at a temperature of 30° C. are cooled down to zero, and at the same time the external pressure upon the air is doubled. What is the volume reduced to, the coefficient of expansion of air for 1° C. being 0·00366?

4. State precisely the law which regulates the connection between the temperature and the pressure and volume of a gas, and show that the statement of this law will be the same, whether we study the increase of volume of a gas whose pressure is constant, or the increase of pressure of a gas whose volume is constant.

5. Explain the phrase "mean coefficient of linear expansion of a substance between 0° and $t°$;" and show that the coefficient of cubical expansion of a metal is very approximately equal to three times its coefficient of linear expansion.

6. What is meant by "the point of maximum density" of water? Sketch and describe an apparatus by which this point may be determined.

7. If a bulb of a thermometer with a fine stem be immersed in hot water, it is sometimes noticed that the mercury falls before it begins to rise. Explain this.

8. Describe some accurate method of determining the coefficient of linear expansion of a steel bar.

If the expansion of steel is $\frac{2}{3}$ that of brass under the same change of temperature, what will be the best arrangement of rods of these metals to form a gridiron pendulum?

9. Why do we generally speak of *degrees* of temperature, but *quantities* of heat?

"If two bodies are in thermal equilibrium with the same body, they are in thermal equilibrium with one another." How would you prove this statement experimentally?

10. Define the coefficient of linear expansion of a substance.

Describe an experiment which shows that the coefficient of expansion of some metals is much greater than that of others.

Explain the construction of a (compensating) chronometer balance wheel.

11. State the law connecting the volume, pressure, and temperature of a given mass of gas. Describe a method of investigating the relation between the pressure and temperature of a quantity of gas whose volume is kept constant.

12. Distinguish between the absolute and apparent expansion of mercury contained in a thermometer.

The coefficient of absolute (cubic) expansion of mercury is ·00018, the coefficient of linear expansion of glass is ·000008. Mercury is placed in a graduated glass tube, and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101 divisions?

13. Why is it necessary to determine the freezing point upon a thermometer before determining the boiling point? In determining the boiling point, why is it necessary to observe the barometer? Is the freezing point affected by the height of the barometer, and if so, why?

14. Distinguish between the coefficients of apparent and of absolute expansion of mercury, and explain how the latter quantity may be determined.

15. How may the relation between the pressure and temperature of a given mass of air at constant volume be determined?

A quantity of air occupies 10 cubic feet at 0° C. and under a pressure of 20 inches of mercury. What will be its volume at 30° C., under a pressure of 1,200 inches of mercury?

16. A glass flask contains, when full at 0° C., 100 c.cm. of mercury. The coefficient of cubical expansion of glass being 0·000026, and that of mercury 0·00018, find the volume at 100° C. of the mercury driven out when the flask and mercury are heated to 100°.

17. Explain accurately what is meant by the statement that the coefficient of expansion of air is $\frac{1}{273}$. The volume of a certain quantity of air at 50° C. is 500 cubic inches. Assuming no change of pressure to take place, determine its volume at −50° C. and at 100° C. respectively.

Advanced.

18. Explain the methods of determining accurately the Freezing and Boiling points of a mercurial thermometer.

19. A barometer with a brass scale which has been adjusted at 0° C. stands at 778 mm. when the temperature is 20° C. What pressure in kilograms per sq. cm. does this indicate?

(Coefficient of linear expansion of brass = 0·0000188 ;
„ cubical „ „ mercury = 0·0001803 ;
Weight of cubic centimetre of mercury at 0° C. = 13·596.)

20. Describe and explain some form of Compensating Pendulum, and show how to find the relative lengths which the different parts must have in order to render the compensation accurate—the coefficients of expansion of the materials being known.

21. Describe and explain fully the method by which the Absolute Expansion of Mercury has been measured.

22. State facts in confirmation of the statement that the expansion of mercury between the temperatures 0° and 100° C. is more uniform than that of water, and that the expansion of air is still more uniform than that of mercury.

23. Assuming that the mean coefficient of expansion of mercury for 1° C. is ·0001815, and that of the glass of a thermometer ·000026, find the reading of such a thermometer, of which the bulb is plunged in water at the temperature of 100° C., while the stem is exposed to air of the temperature of 10° C.

24. Suppose that an English barometer with a brass scale giving true inches at the temperature 62° F., reads 29·5 inches at 45° F.; what is the pressure in true inches of mercury reduced to the specific gravity it has at 32° F.? [The coefficient of linear expansion of brass is 0·00001, and that of the cubical expansion of mercury is 0·0001; both for 1° F.]

25. Describe the method of constructing a standard mercurial thermometer. State and explain the following corrections: (1) on account of change of zero; (2) on account of recent heating; (3) on account of position; (4) on account of temperature of stem. Why is it impossible to have a Water Thermometer?

26. Describe a method of determining the apparent expansion of mercury, and explain how the instrument employed for the purpose may be used as a thermometer.

27. Explain fully what is meant by the mean coefficient of expansion of a substance between assigned limits of temperature.

Describe a method of determining the mean coefficient of expansion of air at constant pressure between 0° C. and 100° C.

28. Describe a method of measuring the coefficient of expansion of a metal rod.

A solid at 0° when immersed in water displaces 500 cubic inches. At 30° C. it displaces 503 cubic inches. Find its mean coefficient of linear expansion between 0° and 30°.

29. Describe experiments which show that in a perfect gas $\frac{p\,v}{t}$ has a constant value, where p, v, and t represent respectively the pressure, volume, and absolute temperature of a given mass of the gas.

What data would be required in order to determine the value of this constant, supposing the gas to be atmospheric air, and its mass one gram?

30. Describe and explain a method of comparing experimentally the expansions of other liquids with that of mercury.

31. Explain how the apparent weight of a body in air varies with its rise of temperature.

A piece of iron, measuring 1000 c.cm., is weighed at 0° C., and again at 100° C. What will be its apparent change in weight?

Coefficient of expansion of air . . = 0·00367 ;
„ „ iron (linear) = 0·000012 ;
Mass of 1000 c.cm. of air at 0° . . = 1·293 grams.

The pressure is supposed to be normal throughout.

32. What is an air thermometer? How is it constructed, and how is it used? What means have we, besides the air thermometer, of measuring temperatures between 400° and 800° C.?

33. At the sea-level the barometer stands at 750 m.m., and the temperature is 7° C., while on the top of a mountain the barometer stands at 400 mm., and the temperature is − 13° C.; compare the weights of a cubic metre of air in the two places.

These barometric readings may be taken as corrected for temperature.

CHAPTER V.

CALORIMETRY.

44. In connection with this subject the following relations should be noticed :—

1. $\mathbf{Q} = \mathbf{m\,s}\,(\mathbf{t}' - \mathbf{t})$ (Art. 39). **Q** denotes the quantity of heat lost or gained by a body of mass **m** and specific heat **s** during change of temperature from **t** to **t′**.

2. $\mathbf{w} = \mathbf{m\,s}$ (Art. 39). **w** denotes the water equivalent of a body of mass **m** and specific heat **s**.

3. **Principle of calculation for problems on method of mixture** (Art. 40). The method of mixture always involves the loss of heat by one portion of the system considered, and a corresponding gain of heat by the remaining portion. If the former portion be represented by A, and the latter by B, the principle of calculation for the method may be stated thus—

$$\left.\begin{array}{l}\text{The loss of heat by A during}\\ \quad\text{the change from its initial}\\ \quad\text{to its final state}\end{array}\right\} = \left\{\begin{array}{l}\text{The gain of heat by B during}\\ \quad\text{the change from its initial}\\ \quad\text{to its final state.}\end{array}\right.$$

In applying this the following points must be attended to : (*a*) There must be no loss or gain of heat from without. (*b*) The total mass of the system must be constant throughout. This condition is really involved in (*a*).

Examples VII.

1. Ten grams of water at 98° C. are poured into a copper vessel, weighing 25 grams and containing 100 grams of water at 6° C. Find the final temperature of the mixture. Specific heat of copper $= 0{\cdot}092$.

Here, if θ denote the final temperature, we have—

The heat lost by the ten grams of water initially at 98° C. $= 10(98 - \theta)$ units.

Heat gained.—(1) By copper vessel $= 25 \times {\cdot}092(\theta - 6)$ units $= 2{\cdot}3(\theta - 6)$ units.

(2) By water in copper vessel $= 100(\theta - 6)$.

Hence, equating we get

$$10(98 - \theta) = 102{\cdot}3(\theta - 6).$$

$$\therefore\ 112{\cdot}3\,\theta = 1593{\cdot}8, \text{ or } \theta = 14{\cdot}2^\circ \text{ C. (nearly).}$$

2. In order to determine the specific heat of silver, a piece of the metal, weighing 21 grams, is heated to 98° C. and then dropped into a calorimeter containing 100 grams of water at 10° C. The final temperature of the mixture is 11° C.; find the specific heat of silver. The water equivalent of the calorimetric apparatus is 3·6 grams.

Here, if s denote the specific heat of silver, we get—

Heat lost by the silver $= 21 \times s \times (98 - 11) = 1827s$ units.

Heat gained—

(*a*) By calorimetric apparatus $= 3{\cdot}6(11 - 10) = 3{\cdot}6$ units.

(*b*) By water in calorimeter $= 100(11 - 10) = 100$ units.

Hence, equating we get—

$$1827\,s = 103{\cdot}6.$$

$$\therefore\ s = \frac{103{\cdot}6}{1827} = 0{\cdot}0567.$$

3. Ten grams of common salt, at 91° C., having been immersed in 125 grams of oil of turpentine* at 13° C., the temperature of the mixture was 16° C. Find, from these data, the specific heat of common salt, supposing no loss or gain of heat to have taken place from without and taking the specific heat of oil of turpentine as 0·428.

Here we have—

$$10 \times s \times (91 - 16) = 125 \times 0{\cdot}428 \times (16 - 13)$$

$$\therefore\ 750\,s = 160{\cdot}5$$

$$\therefore\ s = \frac{160{\cdot}5}{750} = 0{\cdot}214.$$

4. A mass of 200 grams of copper, whose specific heat is 0·095, is heated to 100° C., and placed in 100 grams of alcohol at 8° C., contained in a copper calorimeter, whose mass is 25 grams, and the temperature rises to 28·5° C. Find the specific heat of the alcohol.

The heat lost by the copper

$$= 200 \times {\cdot}095 \times (100 - 28{\cdot}5)$$
$$= (200 \times {\cdot}095 \times 71{\cdot}5) \text{ gram-degrees.}$$

The heat gained—

(*a*) By the calorimeter $= 25 \times {\cdot}095 \times (28{\cdot}5 - 8)$

$$= (25 \times {\cdot}095 \times 20{\cdot}5) \text{ gram-degrees.}$$

(*b*) By the alcohol

$$= (100 \times s \times 20{\cdot}5) \text{ gram-degrees,}$$

where s denotes the specific heat of the alcohol. Then, equating total loss and gain of heat, we have—

$$(200 \times {\cdot}095 \times 71{\cdot}5) = (25 \times {\cdot}095 \times 20{\cdot}5) + (100 \times 20{\cdot}5 \times s),$$

$$1358{\cdot}5 = 48{\cdot}6875 + 2050\,s.$$

$$\therefore\ 2050\,s = 1309{\cdot}3125.$$

$$\therefore\ s = 0{\cdot}6389.$$

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* Water could not be used in this case as common salt is soluble in water. This point should be noticed.

5. The following data were obtained in an experiment for the determination of the water equivalent of a given calorimetric apparatus.

Weight of apparatus	45·623 grams.
" " " + water . . .	224·583 "
Initial temperature of apparatus and water	9° C.
Temperature of hot water	78° C.
Final temperature	13·2° C.
Weight of apparatus after addition of hot water	236·493 "

[*Note.*—These data are given in the order of their determination in an actual experiment.]

Here—

Weight of water in calorimeter $= 224{\cdot}583 - 45{\cdot}623 = 178{\cdot}960$ grams, and weight of hot water added $= 236{\cdot}493 - 224{\cdot}583 = 11{\cdot}91$ grams.

Therefore, if w denote the water equivalent of the apparatus, we have—

$$(178{\cdot}96 + w)\,(13{\cdot}2 - 9) = 11{\cdot}91(78 - 13{\cdot}2),$$
$$4{\cdot}2\,(178{\cdot}96 + w) = 11{\cdot}91 \times 64{\cdot}8,$$
$$751{\cdot}632 + 4{\cdot}2\,w = 771{\cdot}768,$$
$$\therefore\ 4{\cdot}2\,w = 20{\cdot}136.$$
$$\therefore\ w = 4{\cdot}794 \text{ grams.}$$

6. A ball of platinum, whose mass is 200 grams, is removed from a furnace and immersed in 150 grams of water at 0° C. If we suppose the water to gain all the heat which the platinum loses, and if the temperature of the water rises to 30° C., what is the temperature of the furnace? Specific heat of platinum is 0·031.

Here, if T denote the temperature of the furnace, we have—

$$200 \times {\cdot}031(T - 30) = 150 \times 30.$$
$$\therefore\ 6{\cdot}2(T - 30) = 4500.$$
$$\therefore\ 6{\cdot}2\,T = 4500 + 186.$$
$$\therefore\ 6{\cdot}2\,T = 4686.$$
$$\therefore\ T = \frac{4686}{6{\cdot}2} = 755^{\circ}{\cdot}8 \text{ C.}$$

[This example indicates a method of measuring very high temperatures.]

7. A volume of air, at 100° C., whose mass is 30 grams, is passed through a copper worm, immersed in 197 grams of water initially at 10° C., and finally at 13° C. Find the specific heat of air, given that the water equivalent of the calorimetric apparatus is 10 grams, and that the loss of heat by cooling of the calorimeter during the experiment is 9 gram-degree units.

Here the air is not all cooled to the same temperature—that which passes through first is cooled down to 10° C., but as the temperature of the calorimeter rises, the fall of temperature of the air becomes gradually less. If the increase of temperature in the calorimeter be supposed to be uniform, the heat actually lost by the air is approximately equal to that given by supposing the total mass of air to be cooled down to the *mean* temperature of the calorimeter during the time occupied by the experiment.* Hence—

Heat lost by the air—

$$= 30 \times s \left(100 - \frac{10+13}{2}\right) \text{ units} = 30 \times s \times 88{\cdot}5 \text{ units} = 2655\, s \text{ units.}$$

Heat gained by calorimeter and contained water—

$$= 3\,(197 + 10) = 207 \times 3 = 621 \text{ units.}$$

The heat lost by the cooling of the calorimeter was originally derived from the air,

$\therefore$ the total gain of heat $= 621 + 9 = 630$ units.

Hence, equating we get

$$2655\, s = 630$$

$$\therefore\ s = \frac{630}{2655} = 0{\cdot}237.$$

[This example should be studied in connection with Art. **42** to which it is supplementary.]

8. The following data were obtained in an experiment for the determination of the specific heat of shot:

Weight of shot	40·36 grams.
„ „ brass wire basket carrying shot	2·4 „
„ „ calorimetric apparatus	26·4 „
„ „ „ „ + water	114·56 „
Initial temperature of water	10° C.
„ „ „ shot	98° C.
Final „ „ mixture	11°·4 C.

Find the specific heat of the shot, given that the specific heat of brass is ·09, and the water equivalent of the apparatus is 2·8 grams.

9. 280 grams of zinc (specific heat = ·095) are raised to the temperature 97° and immersed in 150 grams of water at 14° contained in a copper calorimeter weighing 96 grams. The specific heat of copper being ·095, what will be the temperature of the mixture supposing that there is no exchange of heat except among the substances mentioned? What is the water equivalent of the calorimeter employed?

10. A copper vessel containing a thermometer is at 12° C.; 50 grams of water at 60° are poured in, and the temperature, after stirring, is

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* It is important to notice that in this experiment the air passes **through** the calorimeter and does not remain in it until the final temperature is reached, but escapes into the open air. It is only when this is the case that the method of calculation adopted in this example is to be used.

found to be 50° : find the thermal capacity, or water equivalent, of the vessel and thermometer.

11. In a determination of the specific heat of a liquid by the method of cooling the following data were obtained :

Weight of calorimeter (copper)	16·24 grams.
„ „ and liquid	27·18 „
„ „ „ water	30·14 „
Time of cooling of liquid from 60° to 55° C.	140 seconds.
„ „ water „ „	330 „

Find the specific heat of the liquid. [The water equivalent of the calorimeter must be determined from its weight, and the specific heat of its material (Art. 39).]

12. Determine the specific heat of copper from the following data :

Weight of copper	16·65 grams.
„ water in calorimeter	49 „
Initial temperature of copper	99°·5 C.
„ „ water and calorimeter	12° C.
Final „ mixture	14°·5 C.
Water equivalent of calorimeter, etc.	2·1 grams.

13. Determine the specific heat of alcohol from the following data :

Weight of copper calorimeter	20·4 grams.
„ „ „ + alcohol	70·5 „
„ „ dropped into calorimeter	10·5 „
Initial temperature of calorimeter and alcohol	10° C.
„ „ „ copper	98° C.
Final „ „ mixture.	12·6° C.

14. Ten grams of sulphuric acid, enclosed in a sealed glass tube weighing 4·3 grams, are heated to 80° C. and dropped into 86 grams of water at 10° C. contained in a copper vessel weighing 15 grams. Find the final temperature of the mixture, the specific heat of sulphuric acid being taken as 0·34.

15. A piece of platinum, weighing 120 grams, is taken from a furnace and at once dropped into 100 grams of water at 10° C., contained in a copper vessel, weighing 21 grams. The final temperature is found to be 37° C.: find the temperature of the furnace.

16. 100 grams of mercury at 250° C. are mixed with 80 grams of mercury at 15° C. in a glass vessel weighing 35 grams. Find the final temperature of the mixture.

17. Regnault found that 100·5 units of heat were required to raise the temperature of unit mass of water from 0° C. to 100° C., and 203·2 units to raise its temperature to 200° C. Find the mean specific heat of water between 0° C. and 100° C., between 100° C. and 200° C., and between 0° C. and 200° C.

18. Equal weights of two liquids of specific heats s' and s'', at temperatures t' and t'' are poured into a glass vessel of mass m, specific heat s, and temperature t. Find the final temperature of the mixture.

19. Equal volumes of two liquids A and B are mixed. Find the final temperature.

Specific gravity of A = 1·8 ; of B = ·56.
Specific heat of A = 0·3 ; of B = 0·6.
Temperature of A = 60° C.; of B = 40° C.

20. One hundred litres of hydrogen, measured at 0° C., are heated in an oil bath to 210° C. and then passed through a calorimeter containing 500 grams of water initially at 10° C., and finally at 21·75° C. Find the specific heat of hydrogen, given that the water equivalent of the calorimetric apparatus = 5 grams, loss of heat by cooling of calorimeter during passage of gas = 3·6 gram-degree units, weight of 1 litre of hydrogen = 0·0896 grams at 0° C.

CHAPTER VI.

CHANGE OF STATE:—LIQUEFACTION AND SOLIDIFICATION.

53. THE following points referred to in the chapter on this subject should be noticed :—

1. If **L** denote the latent heat of fusion of a given substance, then the quantity of heat **absorbed during fusion** by a mass **m** of that substance is represented by **m L**, and the quantity of heat **evolved during solidification** of a mass **m** of the substance is also represented by **m L**.

[In each of these cases the temperature of the substance remains constant during the change of state, but the heat absorbed, or given out, affects the temperature of adjacent substances.]

2. **Bunsen's ice calorimeter.** The formula $s = \frac{882v}{mT}$ should *not* be used for the solution of problems. It is however convenient to know it for the verification of answers obtained. It is also helpful to remember that a decrease of volume, v, in the calorimeter indicates, approximately, the melting of $11v$ grams of ice and a consequent absorption of $11v$ L gram-degree units of heat, where L denotes the latent heat of water. Problems generally contain all necessary data and are best worked out from the given data by the method explained in Art. **52.**

EXAMPLES VIII.

1. Ten grams of ice at $-10°$ C. are mixed with 120 grams of water at 80° C. Find the final temperature of the mixture. (Specific heat of ice $= 0{\cdot}5$ and latent heat of water $= 80$.)

Here, if θ denote the final temperature we have—

Loss of heat by water $= 120(80-\theta)$.

Gain of heat by—

(*a*) Ice during change of temperature from $-10°$ C., to 0° C. $= (10 \times {\cdot}5 \times 10)$ units $= 50$ units.

(*b*) Ice during change of state from ice at 0° C. to water at 0° C. $= 10 \times 80$ units $= 800$ units.

(*c*) Water thus produced during change of temperature from 0° C. to $\theta°$ C. $= 10\theta$.

Hence, equating we get—

$$120(80-\theta) = 50 + 800 + 10\theta.$$
$$\therefore\ 130\,\theta = 8750, \text{ or } \theta = 67{\cdot}3°.$$

2. How many units of heat would cause a mixture of ice and water to contract by 50 c.mm., if 100 c.mm. of water at 0° C. become 109 c.mm. on freezing?

Here, the contraction resulting from the *production* of 100 c.mm. of water is 9 c.mm.

But 100 c.mm. of water = 0·1 c.cm., and therefore weighs, approximately, 0·1 gram.

Hence, a contraction of 9 c.mm. indicates the melting of 0·1 gram of ice and the consequent absorption of $(0{\cdot}1 \times 80) = 8$ gram-degree units of heat.

Therefore a contraction of 50 c.mm. indicates the absorption of $\frac{50 \times 8}{9} = 44\frac{4}{9}$ gram-degree units of heat.

3. Determine the specific heat of a given metal from the following data of an experiment with Bunsen's calorimeter.

Weight of metal dropped into calorimeter .	0·88 grams.
Temperature ,, ,, ,, . .	98° C.
Distance travelled by mercury thread in capillary tube	7·6 mm.
Area of cross section of bore of tube . .	1 sq. mm.
Density of ice	$\frac{11}{12}$

Density is, by definition, the mass of unit volume, *i.e.*—

$$m = v\,d, \text{ or } v = \frac{m}{d}$$

$$\therefore \text{ the volume of 1 gram of water} = 1 \text{ c.cm.}$$

$$\text{and the volume of 1 gram of ice} = \frac{1}{\frac{11}{12}} = \frac{12}{11} \text{ c.cm.}$$

$\therefore$ the diminution of volume resulting from the conversion of 1 gram of ice into 1 gram of water at 0° C. $= \left(\frac{12}{11} - 1\right)$ c.cm. $= \frac{1}{11}$ c.cm.

The decrease of volume in experiment of question $= 7{\cdot}6 \times 1$ c.mm. $= {\cdot}0076$ c.cm.

$$\therefore \text{ mass of ice melted} = \frac{{\cdot}0076}{\frac{1}{11}} = {\cdot}0836 \text{ c.cm.,}$$

and heat absorbed $= {\cdot}0836 \times 80 = 6{\cdot}688$ gram-degree units.

But, heat lost by piece of metal $= {\cdot}88 \times s \times 98 = 86{\cdot}24\,s$.

Hence, equating we get—

$$86{\cdot}24\,s = 6{\cdot}688.$$

$$\therefore s = \frac{6{\cdot}688}{86{\cdot}24} = 0{\cdot}0775.$$

[From formula, $s = \frac{882\,v}{m\,\mathrm{T}}$, we get $s = \frac{882 \times {\cdot}0076}{0{\cdot}88 \times 98} = 0{\cdot}0776\ldots$]

4. If the specific heat of tin is 0·056, and the latent heat 14·25, what quantity of heat is required to raise 6 lbs. of tin from the temperature 208° C. to its melting-point, 238° C., and to melt it?

5. One gram of sal-ammoniac is dissolved in three grams of water, and during the solution as much heat disappears as would raise the temperature of 65 grams of water 1 degree. The specific heat of the solution is 0·75 (nearly): find its temperature, supposing the temperatures of the materials, before they were mixed together, to be 18° C., and that no loss or gain of heat takes place from without?

6. Twenty-five grams of water at 15° C. are placed in the inner tube of a Bunsen's calorimeter, and it is found that 6·8 grams of mercury are drawn in. Assuming the density of mercury as 13·6 and the latent heat of water as 79, determine the density of ice.

[The capillary tube is here supposed to be drawn out to a point which dips under the surface of mercury in a crucible. (Cf. weight thermometer arrangement.) The decrease of volume in the calorimeter is deduced from the difference in weight of the mercury in the crucible before and after the experiment]

7. Determine the latent heat of ice from the following data.

Weight of brass calorimeter (Sp. heat ·09)... ...	35 gm.
,, ,, ,, ,, + water	156 gm.
Initial temperature of water and calorimeter ...	24° C.
Final ,, ,, ,, ...	17° C.
Weight of calorimeter, etc., after addition of ice ...	165 gm.

8. A gram of ice at 0° C. contracts 0·091 c.cm. in becoming water at 0° C. A piece of metal weighing 10 grams is heated to 50° C. and then dropped into the calorimeter. The total contraction is ·063 c.cm.: find the specific heat of the metal, taking the latent heat of ice as 80.

9. Five hundred cubic centimetres of mercury at 56° C. are put into a hollow in a block of ice and it is found that 159 grams of ice are liquefied; find the specific heat of mercury.

10. Ten grams of water at 96° C. are placed in the inner tube of a Bunsen's calorimeter, and it is found that the volume of the contents of the outer portion decreases by 1·09 c.cm.: taking the latent heat of water as 80 what value does this give for the specific gravity of ice?

CHAPTER VII.

CHANGE OF STATE:—VAPORIZATION AND CONDENSATION.

71. In connection with the subject-matter of this chapter the following points should be noticed :—

1. **Dalton's Second Law** (p. 110). If p_1, p_2, p_3 denote the individual pressures due to the several constituents of a mixture of vapours then, if this law be applicable, we have, $P = p_1 + p_2 + p_3$, where P is the total pressure exerted by the mixture. The most familiar example of the application of this law is in the case of air and water vapour. If P′ denote the pressure due to the air alone, and f that due to the vapour, then the total pressure, P, is given by $P = P' + f$.

2. **Latent heat of vaporization.** If **L** denote the latent heat of vaporization of a given liquid then the quantity of **heat absorbed during vaporization** by a mass **m** of that liquid is denoted by **m L**, and the quantity of **heat evolved during condensation** of a mass **m** of the liquid is also denoted by **m L**. (Compare Art. **53, 1.**)

Regnault's formulæ—

$$Q_T = 606{\cdot}5 + 0{\cdot}305T,$$
$$L_T = 606{\cdot}5 - 0{\cdot}695T,$$

should be learnt, and also the general result deduced on p. 126.

Examples IX.

1. A quantity of hydrogen is collected, over water, in a eudiometer tube. The height of the column of water, left in the tube, is 40·8 mm., and its temperature is 15° C. ; find the pressure of the hydrogen in the upper part of the tube. (Take the height of the barometer at 758 mm.) The space occupied by the hydrogen is saturated with aqueous vapour. Hence, with the notation used above—

$$P = P' + f,$$
$$\therefore\ P' = P - f,$$

where P denotes the total pressure in the tube, P′ that due to the hydrogen, and f that due to the aqueous vapour present.

But since 40·8 mm. of water are equivalent to $\frac{40{\cdot}8}{13{\cdot}6} = 3$ mm. of mercury we have that—

$$P = 758 - 3 = 755 \text{ mm.}$$

and, from table on page 148, we get—

$$f = 12{\cdot}7 \text{ mm.}$$
$$\therefore\ P' = 755 - 12{\cdot}7 = 742{\cdot}3 \text{ mm.}$$

2. Find the latent heat of steam at 100° C. from the following data—

Weight of calorimeter	105 gm.
„ „ „ and water	346 gm.
Initial temperature of calorimeter and water .	4° C.
Final „ „ „ „ „ .	24° C.
Weight of calorimeter, etc., at the end of the experiment	354·16 gm.
Water equivalent of calorimetric apparatus .	9 gm.
Height of barometer	752 mm.

(The steam is produced at atmospheric pressure).

Here, from data of question we have—
the weight of water in the calorimeter $= 346 - 105 = 241$ grams;
the weight of steam condensed $= 354{\cdot}16 - 346 = 8{\cdot}16$ grams;
the temperature of the steam $= 99{\cdot}7°$.

Hence—

The **loss of heat**—

(*a*) By the steam during condensation $= m\,L = 8{\cdot}16\,L$ calories.

(*b*) By the water, produced on condensation of the steam, in cooling from 99·7° C. to 24° C. $= 8{\cdot}16(99{\cdot}7 - 24) = 8{\cdot}16\,(75{\cdot}7) = 617{\cdot}712$ calories;

and, the **gain of heat** by the calorimetric apparatus

$$= (241 + 9)(24 - 4) = 250 \times 20 = 5{,}000 \text{ calories.}$$

Therefore, equating we have—

$$8{\cdot}16\,L + 617{\cdot}712 = 5{,}000.$$
$$\therefore 8{\cdot}16\,L = 4382{\cdot}288.$$
$$\therefore L = 537.$$

3. A copper vessel, weighing 100 grams, contains 300 grams of water at 0° C., and 50 grams of ice at 0° C. Find the quantity of steam, at 100° C., that must be blown into the vessel, to raise its temperature and that of its contents to 10° C. (Sp. heat of copper $= 0{\cdot}095$; latent heat of steam $= 537$; latent heat of water $= 80$.)

Let m denote the mass of the necessary quantity of steam. Then the **heat lost**—

(*a*) By the steam during condensation $= 537\,m$ calories.

(*b*) By water so produced in cooling from 100° C. to 10° C.

$$= m(100 - 10) = 90\,m \text{ calories}$$

and, the **heat gained**—

(*a*) By the copper vessel $= 100 \times {\cdot}095 \times 10 = 95$ calories.

(*b*) „ „ water in the vessel $= 300 \times 10 = 3{,}000$ „

(*c*) „ „ ice during liquefaction $= 50 \times 80 = 4{,}000$ „

(*d*) „ „ water, produced by the liquefaction of the ice, in being raised from 0° C. to 10° C. $= 50 \times 10 = 500$ calories.

Hence, equating we get—

$$627\,m = 7595.$$
$$\therefore m = 12{\cdot}113 \text{ grams.}$$

4. A quantity of dry air measures 1,000 c.cm. at 10° C., and 760 mm. pressure. If the same quantity of air is heated to 30° C., and saturated with aqueous vapour at that temperature, what must be the volume of the moist air, in order that the pressure may remain unchanged? (Tension of aqueous vapour at 30° C. = 31·55 mm.)

5. A bubble containing 0·01293 milligrams of air is formed, 136 mm. below the surface of water at 80° C. Find its volume. [1 c.cm. of air weighs 1·293 milligrams at 0° C., and 760 mm. pressure; height of barometer = 750 mm.; remaining datum required may be obtained from table given on p. 111].

6. A bubble of air is formed 68 mm. below the surface of water at 10° C. Find how many bubbles (of air and water vapour) of volume equal to its own, this bubble may give rise to, when the temperature of the water rises to 90° C. Height of barometer = 760 mm.

7. In an experiment with the apparatus of Fig. 30, the height of the mercury column in the barometer tube, A, was 758 mm. Sufficient air was introduced into the vacuum in C. to reduce the height of the column in that tube to 400 mm.; a small quantity of water was then introduced and the space saturated with aqueous vapour at 20° C. Find how far the tube C. must be lowered into the cistern in order to eliminate the pressure of the air, and find the height of the column of mercury in the tube.

8. It is required to condense a mass of unsaturated aqueous vapour existing at 220° C. under a pressure of 5961·66 mm. Give the conditions of pressure and temperature of three different ways of effecting the condensation.

*9. It is found by experiment in a room, where the temperature is 15° C., the dew-point 8° C, and the height of the barometer 750 mm., that a quantity of water in a shallow cylindrical vessel, 10 cm. in radius, loses by evaporation 10 grams of water in 24 hours. If the conditions are such that the formula $m = K\frac{S}{P}(F-f)$ is applicable, find the value of the constant, K, in this case, and apply the formula to find the loss by evaporation from the same vessel when the temperature of the room rises to 20° C., the dew point remaining constant, and the barometric height is 760 mm.

10. The boiling point of water on the top of a mountain is found to be 89° C. What pressure would a barometer there indicate? Express the pressure in dynes per sq. cm.

11. Express the latent heats of steam and water in terms of the degree Fahrenheit.

12. Draw up, from Regnault's formulæ for total and latent heat of steam, a table showing the value of Q and L for the temperatures, 50° C., 60° C., etc. . . . 150° C. [Notice how Q and L change (increase or decrease) with rise of temperature.]

13. Ten grams of steam, at 100° C., are blown into 100 grams of

a mixture of ice and water at 0° C. The final temperature of the mixture is 5° C., find the quantity of ice originally in the mixture.

14. Ten grams of steam at 60° C., are passed into 600 grams of water at 4° C. The final temperature of the mixture is 14·18° C. Find the latent heat of steam at 60° C. Verify your result by Regnault's formula.

[In an experiment such as this some arrangement must be made for keeping the steam at a constant pressure, corresponding to the maximum pressure of aqueous vapour at 60° C. For example, the steam may be condensed in the calorimeter in a thin copper vessel, communicating with a reservoir of air, in which the pressure can be varied at will.]

15. Fifty grams of steam, at 100° C., are passed into a mixture of 100 grams of ice and 200 grams of water at 0° C. Find the rise of temperature produced. The water equivalent of the vessel containing the mixture of water and ice is 15 grams.

16. How many grams of copper at 200° C. must be dropped into a mixture of 20 grams of ice and 20 grams of water at 0° C. to completely convert it into steam at 100° C. The water equivalent of the vessel containing the mixture is 5 grams.

17. Ten grams of ice at −10° C. and 100 grams of water at 10° C. are mixed in a copper vessel weighing 150 grams. Twenty grams of steam at 100° C. are then passed into the mixture. Find the final temperature of the mixture.

18. Find the quantity of heat required to vaporize 10 grams of alcohol at 78·3° C.

19. Twenty grams of ether vapour at 35° C. are passed into 100 grams of ether at 0° C. in a copper vessel weighing 12·5 grams. Find the final temperature of the mixture.

20. Ten grams of melted lead at 332° C. are dropped into a copper vessel surrounded by melting ice. Find the weight of ice melted.

21. Three separate mixtures are made, namely—

(1) Water and snow,
(2) Water and salt,
(3) Snow and salt.

If all the materials were, before being mixed, at 0° C., which mixture will be at the highest temperature and which at the lowest? and why?

22. A glass bottle and a bottle of porous earthenware are both filled with water and exposed to the air side by side. Usually, the water in the earthenware bottle becomes decidedly colder than that in the glass; why is this? If there is little or no difference of temperature, what conclusion may we draw as to the state of the atmosphere? and why?

EXAMINATION QUESTIONS.

Questions set at Various University Examinations.

Elementary.

1. Water is boiled in two tubes, the depth of water in the one being 6 inches, and in the other 24 inches. I find that at a temperature when bubbles of steam rise from the bottom of the one tube they do not rise from that of the other? Explain this.

2. Distinguish between *heat* and *temperature*. Compare the quantity of heat necessary to heat 1 lb. of water from 0° to 1° C., with the quantity necessary to convert 1 lb. of ice at 0° into steam at 100° C. (Latent heat of water = 79·25; latent heat of steam = 536.)

3. What is a calorimeter? Sketch and describe the calorimeter of Laplace, commonly called the Ice Calorimeter.

4. A pound of common salt and a pound of water, both at temperature 15° C., are mixed together, and the salt dissolved. Does any change of temperature take place? Give reasons for your answer.

5. It is found as a result of experiment that 25 grams of copper, at a temperature of 100° C., are just sufficient to melt 2·875 grams of ice at 0°, so that water and copper are finally at 0°. Find from these data the specific heat of copper, taking the latent heat of water to be 80.

6. The specific heat of mercury is said to be $\frac{1}{30}$; what does this mean? If the heat yielded by *one* kilogram of water in cooling down from 100° C. to 0° were employed in heating *ten* kilograms of mercury initially at 20°, to what temperature would the mercury be raised?

7. State the laws of the boiling of liquids. Explain in what way these laws have been made use of to determine the relation between the pressure and temperature of the vapour of water.

8. Define the latent heat of fusion of a substance.

The latent heat of fusion of ice is 79·5. The specific gravity is ·917. Ten grams of metal at 100° C. are immersed in a mixture of ice and water, and the volume of the mixture is found to be reduced by 125 c.mm., without change of temperature. Find the specific heat of the metal.

9. Distinguish between evaporation and ebullition. What condition determines whether a liquid will boil or evaporate? A closed vessel is half full of water and half full of dry air, all at 0° C. and at ordinary

pressure. On heating the vessel the pressure is found to rise to two atmospheres, though the temperature is several degrees below 100° C. Account for this.

10. Define the specific heat and the latent heat of fusion of a substance.

The specific heat of iron is ·113 ; how many lbs. of iron at 250° C. must be introduced into an ice calorimeter in order to produce 2 lbs. of water?

11. State the laws of evaporation. Under what circumstances will a liquid evaporate, and how must the conditions be modified in order that it may boil? What is the dew-point?

12. Define specific heat. What is meant by the latent heat of steam? How does it vary with the temperature at which the steam is produced?

13. Twenty grams of iron at 98° C., (specific heat ·119) are immersed in 80 grams of water at 10° C. contained in a copper vessel whose mass is 15 grams. Find the resulting temperature the specific heat of copper being ·095.

What precautions should be taken to prevent loss of heat during the experiment?

14. What do you understand by a unit of heat?

How would you determine experimentally the number of units of heat required to convert 1 lb. of water at 100° C. into steam at the same temperature? Describe fully the details of the experiment.

15. Describe a method of determining the specific heat of a solid.

How many units of heat would cause a mixture of ice and water to contract by 50 c.mm., if 100 c.mm. of water at 0° C. become 109 c.mm. of ice on freezing?

16. What is meant by the statement that the latent heat of steam is 537? One pound of saturated steam at 160° C. is blown into 19 lbs. of water at 0° C., and the resulting temperature is 32·765° C. Find the latent heat of steam at 160° C.

17. State clearly the distinction between temperature and heat.

Twenty pound-degrees of heat are communicated to a metal vessel weighing 8 lbs., and containing 10 lbs., of water. If the specific heat of the metal be $\frac{1}{10}$, in what proportion will the heat be divided between the water and the vessel, and what will be their rise of temperature?

18. Describe Bunsen's calorimeter. If 100 c.cm. of water in freezing become 109 c.cm. of ice, and the introduction of 20 grams of mercury at 100° C., into a Bunsen's calorimeter cause the end of the column of mercury to move through 74 mm. in a tube 1 sq. mm. in section, find the specific heat of mercury. (The heat required to melt one gram of ice is 80 units.)

19. Explain fully the meaning of the statement that the latent heat of steam is 537. The specific heat of mercury is ·03. A pound of steam at 100° C. is made to pass into a vessel containing 300 lbs. of mercury initially at 0° C., the capacity for heat of the vessel being equal to that of 10 lbs. of water. What will be the temperature of the vessel and contents at the end of the experiment?

20. 200 grams of water at 99° C. are mixed with 200 c.cm. of milk of density 1·03 at 15° C., contained in a copper vessel of thermal capacity equal to that of 8 grams of water, and the temperature of the mixture is 57° C. If all the heat lost by the water is gained by the milk and the copper, what is the specific heat of the milk?

21. A pound of ice at 0° C. is thrown into 6 lbs. of water at 15° C. contained in a copper vessel weighing 3 lbs. and when the ice is melted the temperature of the water is 2° C. Find the latent heat of fusion of ice, the specific heat of copper being 0·095.

22. Define specific heat and describe an experiment by means of which the specific heats of oil and water may be compared.

23. Explain the term *latent heat.* If 25 grams of steam at 100° C. be passed into 300 grams of ice-cold water, what will be the temperature of the mixture, the latent heat of steam being taken equal to 536?

24. Describe experiments illustrating the difference between temperature and heat. In 100 grams of boiling water ($t = 100°$) there are placed 20 grams of ice and the temperature falls to 70° when the ice is just melted. What is the latent heat of fusion of ice, assuming no heat to be lost?

25. Distinguish between saturated and unsaturated vapour.

What is meant by the statement, that when the dew-point is 20° C., the maximum pressure of aqueous vapour in the air is that due to 17·4 mm. of mercury?

26. Define the terms latent heat, specific heat, and capacity for heat. The specific heat of copper is ·095. What is the capacity for heat of 500 grams of copper? If 500 grams of copper are heated to 100° C., and placed in an ice calorimeter, how much ice is melted, the latent heat of fusion of ice being 80?

Advanced.

27. What is meant by "latent heat"? Describe a method of determining the latent heat of steam.

28. Enunciate the law which gives us the relation between the pressure and the volume of a perfect gas of constant temperature. Also enunciate that which gives us the relation between the temperature and the pressure of a perfect gas of constant volume.

Express both laws by one formula, and finally state how the laws are modified in the case of imperfect gases.

29. Define "specific heat." Explain a method of determining the specific heat of a piece of zinc, and describe the apparatus required.

What precautions and corrections are necessary in order to obtain an accurate result?

30. Describe a method of determining the maximum pressure of aqueous vapour at temperatures below 100° C.

31. What conditions influence the temperature at which water freezes and boils?

How does the latent heat of evaporation vary with the temperature at which it takes place? Would you expect the latent heat of fusion to differ with the temperature? Give a reason for your answer.

32. How does the boiling point of a liquid depend on the pressure to which it is exposed? Describe Regnault's method, founded on the variation of boiling-point referred to, of determining the maximum pressure of vapours at various temperatures.

33. Describe and explain the action of a Bunsen calorimeter, whereby quantities of heat are measured by the amount of ice melted by them.

34. Explain carefully the statement that the latent heat of fusion of water is 80. What is the unit in terms of which latent heat is measured? Trace the changes in the temperature and volume of a kilogram of ice at $-5°$ C., to which heat is applied until it is converted into steam.

CHAPTER VIII.

HYGROMETRY.

80. THE principles necessary for the solution of problems connected with hygrometry have been dealt with in Arts. 74—76 of the chapter on this subject. The following points may be again noted :—

1. In calculating the mass of aqueous vapour present in a given volume of air it must be remembered that the total pressure of the mixture is made up of two pressures: (*a*) the pressure of the air; (*b*) the pressure of the vapour present. The latter pressure is equal to the maximum pressure of aqueous vapour at the dew-point, and is the pressure to be employed in calculating the required mass.

2. If the hygrometric state, or relative humidity of air be denoted by h, then—

$$h = \frac{m}{m'},$$

where m denotes the mass of aqueous vapour actually present in the air, and m' denotes the mass of aqueous vapour necessary to saturate the air under the existing conditions.

Also—

$$h = \frac{f}{F},$$

where f denotes the maximum pressure of aqueous vapour at the dew-point, and F denotes the maximum pressure of aqueous vapour corresponding to the temperature of the air.

3. The mass of **1 litre of dry air at 0° C. and 760 mm. pressure is 1·293 grams.**

The mass of **1 litre of aqueous vapour** at any temperature and pressure at which it exists is $\frac{5}{8}$ **of mass of 1 litre of dry air** at the same temperature and pressure.

EXAMPLES X.

1. Two cubic metres of moist air, at 17° C., were drawn through a chemical hygrometer, and 24·12 grams of water were deposited in the tubes. Find the relative humidity of the air.

From above—

$$h = \frac{m}{m'}.$$

Here—

$$m = 24{\cdot}12 \text{ grams},$$

and m' denotes the mass of aqueous vapour necessary to saturate 2 cubic metres (*i.e.* 2,000 litres) at 17° C. The maximum pressure of aqueous vapour at 17° C. = 14·4 mm. (See Table.)

$$\therefore m' = \frac{2{,}000 \times 0{\cdot}808 \times 14{\cdot}4 \times 273}{760 \times 290}$$

$$= 28{\cdot}84 \text{ grams.}$$

$$\therefore h = \frac{m}{m'} = \frac{24{\cdot}12}{28{\cdot}84} = 0{\cdot}837 \text{ nearly;}$$

or, the percentage humidity = 83·7.

2. Find the mass of 1 litre of moist air at 15° C., given, that the dew-point is 10° C., and the barometric height is 759·13 mm.

By (1) above the pressure of the aqueous vapour present = the maximum pressure of aqueous vapour at 10° C = 9·13 mm., and the pressure of the air = 759·13 − 9·13 = 750 mm.

Hence,

$$\text{the mass of the dry air} = \frac{1{\cdot}293 \times 750 \times 273}{760 \times 288}$$

$$= 1{\cdot}2095 \text{ grams;}$$

$$\text{the mass of aqueous vapour} = \frac{{\cdot}808 \times 9{\cdot}13 \times 273}{760 \times 288}$$

$$= 0{\cdot}0092 \text{ grams.}$$

$\therefore$ total weight of litre of moist air
= 1·2095 + 0·0092 = 1·2187 grams.

3. Find the hygrometric state of air at 20° C., the dew-point being 5° C.

The maximum pressure of aqueous vapour at—

$$5^\circ \text{ C.} = 6{\cdot}5 \text{ mm.} = f,$$
$$20^\circ \text{ C} = 17{\cdot}4 \text{ mm.} = F.$$

$$\therefore h = \frac{f}{F} = \frac{6{\cdot}5}{17{\cdot}4} = {\cdot}374\,;$$

or, as a percentage—

$$h = 37{\cdot}4.$$

4. Find the mass of a litre of moist air at 18° C., the dew-point being 5° C., and the barometric height 757·5 mm.

5. The relative humidity of air at 16° C., expressed as a percentage, is 22·5 ; find the dew-point.

6. Two cubic metres of air, at 14° C., are found to contain 18·56 grams of moisture. Find the dew-point and relative humidity of the air.

7. Twenty litres of moist air, at 15° C., are drawn through a chemical hygrometer, and found to contain 0·1863 grams of moisture. What is the hygrometric state of the air?

8. Two hundred c. cm. of hydrogen, measured at 15° C. and 754·68 mm. pressure, are collected over water. Find the mass of the hydrogen present. (1 litre of hydrogen at 0° C. and 760 mm. pressure weighs 0·0896 gram.)

9. The dew-point of air at 20° C. is 8° C. Find the relative humidity and the mass of aqueous vapour present in 1 litre of this air.

10. Find the mass of dry air present in 10 litres of moist air, at 10° C. and 760 mm. pressure, the dew-point of the air being 5° C.

CHAPTER IX.

TRANSMISSION OF HEAT.

91. In connection with this subject it may be useful to summarise the following points involving quantitative relations :—

1. **Absolute conductivity.** In Art. 84, we have the important relation expressed by—

$$H = k\,\frac{A\theta t}{x}. \qquad (1)$$

Also, deduced from this, we have—

$$k = \frac{Hx}{A\theta t}. \qquad (2)$$

[Of these (1) only should be learnt; from it, (2) can be obtained when required.]

*2. If k denote the absolute conductivity of a given substance, then $\frac{k}{sd}$ denotes the diffusivity of that substance. Hence, if κ denote diffusivity, we have—

$$\kappa = \frac{k}{sd}. \qquad (3)$$

Examples XI.

Reference should be made where necessary to the table of conductivities on page 151.

1. Find the quantity of heat that will be transmitted, in 1 hour, across a plate of copper 1 sq. metre in area and 5 cm. thick, the difference between the temperatures of its faces being 10° C.

From (1) above we have—

$$H = k\,\frac{A\theta t}{x}.$$

Adopting the C. G. S. system of units, we have—

$k = 1$ (see table, p. 206),
$A = 1$ sq. metre $= 10000$ sq. cm.,
$\theta = 10^\circ$ C.,
$t = 1$ hour $= 3600$ seconds,
$x = 5$ cm.

$$\therefore\ H = \frac{10000 \times 10 \times 3600}{5}$$

$$= 72000000 \text{ gram-degrees.}$$

2. It is found that 9162000 gram-degrees of heat are transmitted, per minute, across a sheet of silver, 100 sq. cm. in area and 1 mm. thick, with a difference between the temperatures of its faces of 100° C. Find, in C. G. S. units, the absolute conductivity of silver.

From (1) above we have—

$$H = k\,\frac{A\theta t}{x}\,;$$

and therefore—

$$k = \frac{Hx}{A\theta t}$$

Here—

$H = 9162000$ gram-degrees,
$x = 0{\cdot}1$ cm.,
$A = 100$ sq. cm.,
$\theta = 100°$ C.,
$t = 60$ seconds.

Substituting, we get—

$$k = \frac{9162000 \times 0{\cdot}1}{100 \times 100 \times 60} = 1{\cdot}527.$$

3. Sixty kilogram-degrees of heat are transmitted, in 1 minute, across a plate of copper, 100 sq. cm. in area and 1 cm. thick, and having 10° C. difference of temperature between its faces. Find the conductivity of copper in units involving the kilogram, metre, hour, and degree Centigrade.

Here, as in Example 2—

$$k = \frac{Hx}{A\theta t}\,;$$

and, in the given units—

$H = 60$ kilogram-degrees,
$x = 1$ cm. or $0{\cdot}01$ metre,
$A = 100$ sq. cm. $= 0{\cdot}01$ sq. metre,
$\theta = 10°$ C.,
$t = 1$ minute $= \frac{1}{60}$ hour.

Therefore, on substituting—

$$k = \frac{60 \times 0{\cdot}01 \times 60}{0{\cdot}01 \times 10} = 360$$

*4. The specific heat of copper is 0·095, and its density is 8·9; find, in C. G. S. units, the measure of its diffusivity. Find the thickness of a plate of copper, that would be raised in temperature, through 1° C., by the heat transmitted, in unit time, through another copper plate of the same area and 1 cm. in thickness, with a difference of temperature of 1° C. between its faces. Also find the number of degrees rise of temperature produced in a plate of copper of the same area and 1 cm. in thickness, by the same flow of heat.

From (3) we get—

$$\kappa = \frac{k}{sd}.$$

Here—

$$k = 1$$
$$s = 0{\cdot}095$$
$$d = 8{\cdot}9.$$

$$\therefore\ \kappa = \frac{1}{0{\cdot}095 \times 8{\cdot}9} = \frac{1}{{\cdot}8455} = 1{\cdot}183.$$

Let A denote in sq. cms. the area of the plates referred to in question ; then the heat transmitted is given by—

$$\text{H} = k\ \frac{\text{A}\theta t}{x} = 1\ \frac{\text{A} \times 1 \times 1}{1} = \text{A gram-degrees.}$$

If y denote the thickness of the copper plate which will be raised 1° C. by this quantity, A, of heat, then, by Art. 44, 1, we have—

$$\text{A} = m.s.1 = \text{A}y d.s.1 = \text{A}yds.$$

$$\therefore\ 1 = yds. \qquad \therefore\ y = \frac{1}{sd} = 1{\cdot}183 \text{ cm.}$$

Therefore the measure of this thickness is also the measure of the diffusivity. Further, let n denote the number of degrees rise produced in a plate of area A and 1 cm. thick, by this quantity, A, of heat, then, as above—

$$\text{A} = msn = \text{A}.1.dsn = \text{A}dsn.$$

$$\therefore\ 1 = dsn. \qquad \therefore\ n = \frac{1}{sd} = 1{\cdot}183^\circ \text{C.}$$

Therefore diffusivity is also measured by this rise of temperature ; hence the term **thermometric conductivity.**

5. Peclet has stated that the quantity of heat which passes, in an hour, through a plate of lead 1 sq. metre in area and 1 cm. thick, with a difference of 1° C. between the temperature of its surfaces, is 1383 kilogram-degrees. What value does this give for the absolute conductivity of lead in the C. G. S. system?

6. The absolute conductivity of copper in the C. G. S. units is 1; how many heat-units will pass, per minute, across a plate of copper, 1 metre long, 1 metre broad, and 5 cm. thick, when its opposite faces are kept at temperatures differing by 100° C.?

7. The thermal conductivity of felt, in C. G. S. units, is 0·000087; find the quantity of heat that is transmitted, in one hour, through a layer of felt 1 cm. in thickness and 20 sq. cm. in area, when its opposite faces are kept at temperatures differing by 20° C.

8. Calculate the quantity of heat lost, per hour, from each square metre of the surface on an iron steam boiler 0·8 cm. in thickness, when the temperature of the inner surface of the boiler is 120° and that of the outer surface 119·5°, the coefficient of conductivity of iron being 11·5 (referred to 1 cm. as unit of length, 1 minute as unit of time. and the quantity of heat required to raise the temperature of the gram of water from 0° to 1° C. as unit of heat).

9. A square metre of a substance, 1 cm. thick, has one side kept at 100° C., and the other, by means of ice, at 0° C. In the course of 10 minutes one kilogram of ice is melted by this operation. Calculate the conductivity of the substance, assuming the latent heat of water to be 80.

10. The mean temperature of the earth at a depth of 972 feet, being 23° C., and 14° C. at the surface, and the average estimated loss of heat per square foot of surface in 27 years being 4·5 units of heat, find the coefficient of conductivity per cubic foot per hundred years.

*11. Find, in C. G. S. units, the diffusivity of iron, given that the density of iron is 7·5 and its specific heat 0·114.

Express, in words, two other quantities having the same *measure*, and show, as in Ex. 4, that such is the case.

12. Express the conductivity of copper in units involving the pound, foot, second, and degree Fahrenheit.

EXAMINATION QUESTIONS.

Questions set at Various University Examinations.

Elementary.

1. A source of heat is applied equally to the extremities of two similar bars, one of copper and one of iron; and on each bar there is a piece of phosphorus two inches from the source of heat. That on the copper takes fire first. Does this experiment entitle us to conclude that copper has greater thermal conductivity than iron? If not, explain in what respects it is deficient as a proof.

2. How would you compare the thermal conductivities of brass and copper? Two equal cylinders, one of iron and the other of bismuth, are covered with wax and simultaneously placed on end on a hot metal plate. At first the melting of the wax advances more rapidly on the bismuth bar; but after it has melted about an inch up both cylinders, the melting advances the more rapidly on the iron bar. Account for these phenomena.

3. Define latent heat, specific heat, capacity for heat, coefficient of cubic expansion, and thermal conductivity. How would you determine the capacity for heat of a copper vessel?

4. Define the dew-point.

A cylinder which we may suppose impervious to heat is closed by a piston, and contains steam, with a little water, at 100° C. The piston is suddenly depressed so as to compress the steam. State fully what happens.

5. Describe an experiment which shows that water is a very bad conductor of heat.

6. What are the laws of pressure in a mixture of gases and vapours? Explain the principle of, and describe the method of using, the wet and dry-bulb hygrometer.

7. Describe and explain the method of using some form of dew-point hygrometer, and show how to determine the humidity of the air by means of it.

8. Explain the expression "tension of aqueous vapour." How is the pressure of the aqueous vapour in the atmosphere connected with the dew-point? Describe some method of determining the dew-point.

9. A building is heated by hot-water pipes. How does the heat get from the furnace of the boiler to a person in the building? What would be the effects on the temperature of the more distant parts of the building of coating the pipes near the boiler (*a*) with woollen felt, (*b*) with dull black lead?

Advanced.

10. Define the dew-point. Show how to find it by means of a Daniell's hygrometer; and explain the principles involved in the experiment.

11. Describe the hygrometers of Daniell and Regnault, and state what you conceive to be the advantage which the latter has over the former.

12. Define "hygrometric state," and describe and explain the use of some form of condensation hygrometer.

What is the hygrometric state in a room of temperature 20°, in which the dew-point is found to be 11°?

[Maximum tension of aqueous vapour at $20° = 17{\cdot}39$ mm.
" " " " $11° = 9{\cdot}79$ mm.]

13. Give some account of Regnault's determinations of the maximum pressure of vapour at temperatures below the boiling point.

14. Given two similar bars of gold and silver, describe an experiment by which their relative thermal conductivities may be determined.

15. One hundred cubic centimetres of oxygen, saturated with water, are collected at a pressure of 740 mm. and a temperature of 15°C. Find the volume of dry oxygen at 0° and 760 mm., having given that the maximum pressure of aqueous vapour at 15° is 12·7 mm.

*CHAPTER X.

THE MECHANICAL EQUIVALENT OF HEAT.

The first law of thermodynamics. If **W** denote a definite quantity of **work**, **H** the equivalent quantity of **heat**, and **J** the mechanical equivalent of heat, then the relation—

$$\mathbf{W} = \mathbf{J}\,\mathbf{H}$$

expresses the first law of thermodynamics.

Work done by a gas during expansion at constant pressure. If p denote the external pressure *per unit area*, and v the change of volume effected during expansion, then the external work done is given by—

$$w = p\,v.$$

Efficiency of an engine. If H denote the heat received from the *source*, h the heat converted into useful work, and e the efficiency of the engine, then—

$$e = \frac{h}{H}.$$

Numerical Details of One of Joule's Experiments on the Friction of Water.

1. **Heat** :—

Weight of water in calorimeter =	93229·7	grains.
Water equivalent of ,,	2430·2	,,
,, ,, ,, paddle, etc.	1810·3	,,
Total	97470·2	,,

Rise of temperature . . .	0·563° F.
Correction for radiation . .	0·013° F.
Corrected rise of temperature =	0·576° F.

∴ **Heat generated = 7·8423 pound-degrees (*Fahrenheit*).**

2. **Work** :—

Weights employed weighed .	406152	grains.
Mean weight required to balance friction	2837	,,
Effective weight =	403315	,,

Total height of fall (sum of 20) 1260·248 inches.
Velocity of weights on reaching ground was 2·42 in per sec.; hence energy lost is equivalent to that acquired by weights falling through . ·152 „

Effective fall of weights = 1260·096 „

$\therefore$ **Work done = 6050·186 foot-pounds.**

$$\therefore \mathbf{J} = \frac{\mathbf{W}}{\mathbf{H}} = \frac{6050 \cdot 186}{7 \cdot 8423} = \mathbf{773 \cdot 64.}$$

J = 772 (*pound, foot, degree Fahrenheit*).
= 1390 („ „ „ *Centigrade*).
= 424 (*gram, metre* „ „).
= 42400 (*gram, cm.* „ „).
= $4 \cdot 16 \times 10^7$ (*erg.* „ „).

EXAMPLES XII.

1. A mass of 10 pounds falls to the ground from a height of 695 feet. Assuming that it does not rebound, find the heat liberated by its impact on the ground.

Here, work done = 6950 foot-pounds. Taking J = 1390, the heat equivalent of this work is $= \frac{6950}{1390} = 5$ *gram-degrees.*

2. An engine consumes 3 pounds of coal per horse-power per hour. The heat developed by the combustion of 1 pound of coal is capable of converting 15 pounds of water at 100° C. into steam at 100° C. Find the efficiency of the engine.

1 pound of coal produces 15×537 *pound-degrees* of heat.
$\therefore$ 3 pounds „ „ produce $3 \times 15 \times 537$ „ „ „ „

That is, the heat absorbed by the engine per hour is—

$3 \times 15 \times 537$ *pound-degrees.*

And the work performed by the engine per hour is equivalent to—

$\frac{33000 \times 60}{1390}$ *pound-degrees* of heat.

Therefore, the efficiency of the engine is given by—

$$e = \frac{h}{H} = \frac{33000 \times 60}{1390 \times 3 \times 15 \times 537} = 0 \cdot 0589 = 5 \cdot 89 \text{ per cent.}$$

3. Show that the work done by a gas during expansion under constant pressure, for one degree rise in temperature, is the same for all pressures and temperatures.

Let V denote the volume of the gas, T its absolute temperature, and P the external constant pressure.

The increase of volume for 1° rise in temperature is evidently $\frac{V}{T}$, and the work done during the expansion is therefore given by $\frac{P\,V}{T}$. But $\frac{P\,V}{T}$ is constant for all pressures and temperatures. (Art. **34.**)

4. Show that the difference between the two thermal capacities per unit volume is the same for all gases at the same pressure and temperature.

[Employ the method of Ex. 3.]

5. Assuming that the mass of a cubic foot of steam at 100° C. and 760 mm. pressure is 240 grains, find what fraction of the latent heat of steam is consumed in doing external work, *i.e.* in lifting the atmosphere.

6. An engine consumes 40 pounds of coal of such calorific power that the heat developed by the combustion of 1 pound is capable of converting 16 pounds of water at 100° C. into steam at the same temperature, and during the process the engine performs 16,000,000 foot-pounds of work. What percentage of the heat produced is wasted?

7. What is a heat-engine? and what is a reversible heat-engine? What condition must be fulfilled respecting the passage of heat to and from the working substance in order that a heat-engine may be reversible? What is the efficiency of an engine which consumes 28 pounds of coal in drawing a train one mile against a resistance equal to the weight of 1½ tons, the calorific power of the coal being such that 1 pound is capable of converting 16 pounds of boiling water into steam at the same temperature?

8. How can the amount of work done against external pressure during change of volume be expressed numerically?

1 gram of air is heated under constant pressure from 0° to 10° C.; determine the work, either in *ergs* or in *centimetre-grams*, due to the expansion.

[Co-efficient of expansion of air $\frac{1}{273}$. Volume of 1 gram of air at 0°, under pressure of one million dynes per square centimetre, = 783·8 cubic centimetres. Or, 1 cubic centimetre of air at 0° under pressure of 76 cm. mercury = 0·001293 gram; 1 cubic centimetre mercury at 0° = 13·596 grams; $g = 981$ (centim., seconds).]

9. Distinguish between the specific heat of air under constant pressure and its specific heat under constant volume. Show how, from a knowledge of the two specific heats of air, together with its density at given pressure and temperature, the value of the mechanical equivalent of heat may be computed.

10. Given that the ratio of the two specific heats of air is 1·41, and that the work done during expansion, at normal pressure, by 1 gram of air when its temperature is raised from 0° C. to 1° C. is 2926 centimetre-grams; find the value of the two specific heats.

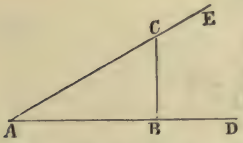
LIGHT.

CHAPTER I.

RECTILINEAR PROPAGATION OF LIGHT. PHOTOMETRY.

14. THE calculations connected with the subject-matter of the first three chapters of the text are simple applications of the elements of geometry or algebra to the principles there explained, and need no further illustration than is afforded by the worked examples given below.

Note.—In Chapter III. we have made use of the term *cosine*, and in succeeding chapters it will be necessary to make frequent use of the term *sine*. Hence for the convenience of the reader we shall now explain these terms.



Let D A E represent a plane angle. From any point C, in A E, draw C B perpendicular to A D, and cutting A D in B. Now the length of B C, for a given position of C, evidently depends on the magnitude of the angle D A E, but it gives no indication of this magnitude unless the position of C be defined. For this purpose the ratio $\frac{BC}{AC}$ may be considered, and it can be shown geometrically that wherever C be taken on A E this ratio is constant, and is definitely related to the magnitude of the angle B A C. Similarly the ratio $\frac{AB}{AC}$ is constant and bears a fixed relation to the magnitude of B A C. The ratio $\frac{BC}{AC}$ is called the *sine* of B A C, and the ratio $\frac{AB}{AC}$ is called the *cosine* of B A C. In the right-angled triangle B A C, *considered with reference to the angle* B A C, the side B C is called the *perpendicular*, the side A B is called the *base*, and A C is called the *hypotenuse*. Hence, in general terms—

$$\text{sine } BAC = \frac{\text{perpendicular}}{\text{hypotenuse}} = \sin BAC.$$

$$\text{cosine } BAC = \frac{\text{base}}{\text{hypotenuse}} = \cos BAC.$$

The reader should deduce geometrically the values of these ratios for angles of 30°, 45°, and 60° These will be found to be—

$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2}.$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}. \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}.$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}. \qquad \cos 60^\circ = \frac{1}{2}.$$

Examples I.

1. In a pinhole camera the distance from the aperture in front, to the screen at the back, is 18 inches. Find the relative dimensions of the representation on the screen of an object placed 6 feet in front of the camera.

In Fig. 6, treating the pencils from A and B to A′ and B′ respectively, as lines we see that the triangles A O B and A′ O B′ are equiangular, and therefore similar (Euclid vi. 4).

$$\therefore \frac{A\,B}{A'\,B'} = \frac{C\,O}{O\,C'}.$$

Here, C O = 6 feet and O C′ = $1\frac{1}{2}$ feet.

$$\therefore \frac{A\,B}{A'\,B'} = \frac{C\,O}{O\,C'} = \frac{6}{1\frac{1}{2}} = 4.$$

$$\therefore \quad A\,B = 4A'\,B'.$$

2. A circular uniform source of light, 2 inches in diameter, is placed at a distance of 10 feet from a sphere 2 inches in diameter. Calculate, approximately, the diameters of the umbra and penumbra cast on a screen 5 feet beyond the sphere.

Here, in Fig. 9—

S S′ = 2 inches; O O′ = 2 inches; S O = 10 feet; O u = 5 feet.
Diameter of umbra = $u\,u$ = O O = .2 inches.

Diameters of penumbra $\begin{cases} \text{Internal} = u\,u = 2 \text{ inches} \\ \text{External} = p\,p = 4 \text{ inches} \end{cases}$; for, from the triangles O $u\,p$ and O S S′, we have, by Euclid vi. 4—

$$\frac{u\,p}{S\,S'} = \frac{u\,O}{O\,S} = \frac{5}{10} = \frac{1}{2}$$

But S S = 2 inches.

$$\therefore \frac{u\,p}{2} = \frac{1}{2}, \text{ or } u\,p = 1 \text{ inch.}$$

$$\therefore \quad p\,p = u\,u + 2\,u\,p = 2 + 2 = 4 \text{ inches.}$$

✓ 3. The intensity of illumination of a screen placed 6 feet from a given source of light is denoted by I. Find the intensity when the distance of the screen is increased to 9 feet.

Let I′ denote the required intensity. Then, by Art. 8—

$$\frac{I'}{I} = \left(\frac{6}{9}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}.$$

That is, $I' = \frac{4}{9} I$.

✓ *4. A small screen is held 6 feet from a source of light, in such a position that the light is incident on it normally. It is then removed to a distance of 10 feet and turned round, so that the light is incident on its surface at an angle of 60°. Compare the intensities of illumination of the screen in the two cases.

Let I and I′ denote the intensities of illumination for the first and second cases respectively. Then, by Arts. 8, 9, the intensity of illumination varies *inversely* as the squares of the distances, and *directly* as the cosine of the angles of incidence. That is—

$$\frac{I}{I'} = \left(\frac{10}{6}\right)^2 \cdot \frac{\cos 0^\circ}{\cos 60^\circ}.$$

Now—

$$\cos 0^\circ = 1, \text{ and}$$
$$\cos 60^\circ = \tfrac{1}{2}.$$

$$\therefore \frac{I}{I'} = \left(\frac{5}{3}\right)^2 \times 2 = \frac{50}{9}.$$

✓ 5. Two sources of light, A and B, when placed respectively 8 and 10 feet from a screen produce the same intensity of illumination of its surface. Compare the illuminating powers of A and B.

Here, by Art. 8—

$$\frac{\text{Illuminating power of A}}{\text{Illuminating power of B}} = \left(\frac{8}{10}\right)^2 = \frac{16}{25}.$$

✓ 6. The intensities of two sources of light, A and B, which are placed 10 feet apart, are as 4 : 9. Find at what points on the line joining them the intensity of illumination is the same.

Let x denote, in feet, the distance of either of the required points from A.

Then—

$$\left(\frac{x}{10-x}\right)^2 = \frac{4}{9} = \left(\pm\frac{2}{3}\right)^2$$

$$\therefore \frac{x}{10-x} = \pm\frac{2}{3}.$$

That is—

$$\begin{aligned} 3x &= 20-2x \\ 5x &= 20, \quad \text{and } x = 4 \text{ feet;} \\ \textit{or}\ 3x &= -20+2x. \\ \textit{and}\ \ x &= -20. \end{aligned}$$

That is, there is equality of illumination at a point between A and B, 4 feet from A and 6 feet from B; also at a point 20 feet from A on the side remote from B. [That is, the line A B is divided internally and externally in the ratio 2 : 3.]

7. A circular uniform source of light, 10 cm. in diameter, is placed 1 metre in front of a spherical opaque body 5 cm. in diameter. Find the shortest distance from the latter at which a screen may be placed so as to have no umbra in the shadow cast upon it; also find the diameter of the penumbra in this position [Fig. 10].

8. A luminous sphere, 5 cm. in diameter, is placed 150 cm. from a disc of wood of 25 sq. cm. area. Find the dimensions of the umbra and penumbra cast on a screen 50 cm. behind the disc of wood. The line passing through the centre of the luminous sphere and the disc is perpendicular to the latter and to the screen.

9. In Fig. 6, C O = 3 metres, O C′ = 20 cm., and the diameter of the aperture at O is 1 mm. Find the area of the circular spot of light at C′ due to the pencil of light coming from C. If A B = 2 metres, find also the length of A′ B′.

10. The intensities of two sources of light are in the ratio 9 : 16. Find the ratio of the distances at which they must be placed from a screen, in order to produce on it the same intensity of illumination.

11. The lines joining the points A, B, and C form an equilateral triangle. D is the middle point of B C. A screen is placed at A with its surface parallel to B C. Lights placed at B, C, and D are found to equally illuminate the screen at A; compare their illuminating powers.

12. In Foucault's photometer (Fig. 12) $E L_1 : E L_2 :: a : b$. Find the relative intensities of L_1 and L_2.

13. In Rumford's photometer (Fig. 13) $L_1 s$ is found to be 115 cm., and $L_2 s$ to be 201 cm. Compare the illuminating powers of L_1 and L_2.

14. The intensities of two sources of light are in the ratio 4 : 9. If these sources are 200 cm. apart, where would a Bunsen's photometer be in accurate adjustment between them?

15. The distance between two incandescent lamps of 16 and 25 candle-power respectively is 6 feet. Show that there are two positions, on the line joining the lamps, at which a screen may be placed so as to receive equal illumination from each lamp; and determine these positions.

CHAPTER II.

REFLEXION AT PLANE SURFACES.

28. All problems on reflexion at plane surfaces are, more or less, geometrical deductions, involving a knowledge of the laws of reflexion in addition to the usual geometrical propositions.

The results of Art. 27 are not of very great importance, but the simple case where θ is an aliquot part of 360° should be remembered. In this case the number of images formed is $\left(\frac{2\pi}{\theta} - 1\right)$.

Note.—In preparation for the work of the next chapter the reader should notice the following points :—

1. The results of Euclid vi. 3, A, and 4.

2. The meaning of the terms *infinite* and *infinity*. A quantity becomes *infinite* when its value becomes greater than any value we can assign to it. If the value of any quantity q is infinite, this is expressed by writing $q = \infty$.

The term *infinity* will be best understood from its use in the statement that parallel straight lines meet at infinity. If any straight line O A be produced to A′, in the direction O A, until it is of infinite length, the point A′ will be at infinity.

3. Consider the ratio $\frac{a}{x}$. If x becomes infinite, the ratio becomes $\frac{a}{\infty}$, and the value of this expression is zero. That is—

$$\frac{a}{\infty} = 0$$

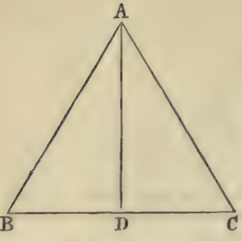
where a is *any* finite quantity.

4. The sine of any angle is equal to the sine of its supplement. That is—

$$\sin a = \sin (180 - a).$$

This is readily seen from a figure.

5. In any triangle the sides are proportional to the sines of the opposite angles.



In the triangle A B C we have—

$$\sin A B C = \frac{A D}{A B}$$

$$\sin B C A = \frac{A D}{A C}. \qquad \text{(Art. 14, note.)}$$

$$\therefore \frac{\sin A B C}{\sin B C A} = \frac{A C}{A B}.$$

Similarly—

$$\frac{\sin B C A}{\sin C A B} = \frac{A B}{B C}, \text{ and}$$

$$\frac{\sin C A B}{\sin A B C} = \frac{B C}{C A}.$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

That is—

Sin A B C : sin B C A : sin C A B :: C A : A B : B C.

Q.E.D.

Examples II.

1. A ray of light starts from A, meets a plane reflecting surface at M, and is reflected to B. Prove that A M B is the shortest possible path from A to B by way of the mirror.

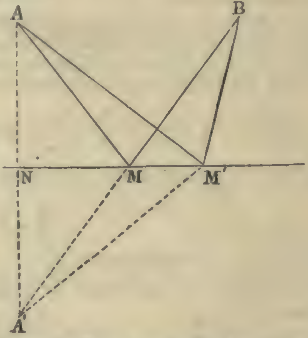
If A M B be not the shortest path, let *any* other path A M′ B be shorter. Draw A N A′ normal to the mirror, and produce B M to meet A A′ in A′.

Then, since A N = A′ N we have, by Euclid i. 4, A M = A′ M and A M′ = A′ M′.

But A′ M′ + M′ B > A′ B > A′ M + M B (Euc. i. 20).

∴ A M′ + M′ B > A M + M B.

Q.E.D.



✓ 2. An object is placed between two mirrors inclined at an angle of 60°; find how many images are formed, and show that the images formed in the angle vertically opposite that contained by the mirrors are coincident. (The conditions of this question are represented in Fig. 26.)

Since 60° is an aliquot part of 360°, we have, for the number of images formed—

$$n = \left(\frac{2\pi}{\theta} - 1\right) = \left(\frac{360}{60} - 1\right) = 5.$$

Also, A_1''' and A_2''' are the images to be shown coincident. For this purpose we must prove $A\,O\,A_1''' + A\,O\,A_2'''$ (measured in *opposite* directions) equal to 360°.

If $A\,O\,M_1 = \alpha$ and $A\,O\,M_2 = \beta$, then, by the method of Art. 27, we have—

$$\begin{aligned} A\,O\,A_1''' &= 2\alpha + 2(60) = 120 + 2\alpha \\ A\,O\,A_2''' &= 2\beta + 2(60) = 120 + 2\beta. \\ \therefore\ A\,O\,A_1''' + A\,O\,A_2''' &= 240 + 2(\alpha + \beta) \\ &= 240 + 120 = 360. \end{aligned}$$

✓ 3. What must be the angle between two plane mirrors in order that a ray incident parallel to one of them may, after two reflexions, be parallel to the other?

Let α denote the angle between the mirrors, then, after two reflexions, the deviation produced $= 2\alpha$. But the deviation required by question $= 180 - \alpha$.

$$\begin{aligned} \therefore\ 2\alpha &= 180 - \alpha; \\ \therefore\ 3\alpha &= 180; \\ \therefore\ \alpha &= 60. \end{aligned}$$

✓ 4. A small object is placed between two parallel mirrors as in Fig. 23. The distance between the mirrors is 6 inches, and the object is placed 2 inches from one of them. Find the distances between the corresponding members of the two series of images formed; also the distances between the odd members of each series, and between the even members of each series.

✓ 5. The sun is 30° above the horizon, and you see his image in a tranquil pool. What, in this case, is the angle of incidence and reflexion?

✓ 6. A man, 6 feet high, sees his image in a plane mirror hung vertically. The top of the mirror being 6 feet from the ground, determine its smallest length in order that the man may see his full-length image in it.

✓ 7. Find the deviation produced by reflexion at a plane mirror, when the angle between the incident and reflected rays is 80°.

✓ 8. The angle between two mirrors is 10°. A ray parallel to one mirror strikes the other. Show that it will return along the same track after several reflections. How many times is it reflected in all?

9. Show, that if a ray of light be incident at any angle, on one of two mirrors inclined at right angles to each other, then the ray is reflected from the second mirror in a direction parallel to its original direction.

10. A mirror revolves about a horizontal axis parallel to its surface. Show how to find if the reflecting surface is accurately parallel to the axis of revolution.

CHAPTER III.

REFLEXION AT SPHERICAL SURFACES.

37. THE formulæ of importance in the chapter on this subject are—

$$1. \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{r}. \qquad \text{(Art. 32.)}$$

Distances measured from the pole of the mirror.

$$*2. \qquad x\,x'=f^2.$$

Distances measured from the focus.

$$3\ (a). \quad \frac{\text{Image}}{\text{Object}}=\frac{v}{u}.$$

Distances measured from the pole of the mirror.

$$3\ (b). \quad \frac{\text{Image}}{\text{Object}}=\frac{c'}{c}.$$

Distances measured from the centre of curvature.

$$3\ (c). \quad \frac{\text{Image}}{\text{Object}}=\frac{f}{u-f}.$$

Distances measured from the pole of the mirror.

Distances measured in a direction **opposed** to that of the incident light are considered **positive**, and those measured in the **same** direction as the incident light are considered **negative**.

This convention applies to all cases, wherever the distance considered may be measured from. In applying the above formulæ the following points must be noticed :—

1. On substituting a numerical value for any of the symbols, the sign of the former must always be attached.

For example, if in formula (1), $u=6$ and $v=-8$, then, on substitution, we get—

$$\frac{1}{-8}+\frac{1}{6}=\frac{1}{f}=\frac{2}{r}.$$

$$\therefore \frac{1}{24}=\frac{1}{f}.$$

$$\therefore f=24 \text{ and } r=48.$$

2. In applying a formula to determine one of the involved distances, the others being known, no sign must be given to the unknown distance. Thus, in the above example, no sign is at first given to f;

but the result, when worked out, shows it to be positive—that is, the mirror is concave.

3. When distances are measured from the pole of the mirror [formulæ 1, 3 (*a*), and 3 (*c*)], the radius of curvature and focal length are **positive** for a **concave** mirror, and **negative** for a **convex** mirror. This is in accordance with the sign convention given above, and needs special notice only as a reminder.

4. Always draw a fairly accurate figure representing given conditions. This prevents mistakes as to sign.

Formulæ 1 and 3 (*a*) are the most important.

Formulæ 3 (*a*) and 3 (*b*) should be learnt *in words* (Art. **34**); 3 (*c*) is not important, but is sometimes very convenient. The different forms of formula 3 may be remembered by noticing that "image" and "v" are associated, as are also "object" and "u."

5. Sign need not be considered in connection with formula 3 if the ratios be learnt *in words*. But if learnt as formulæ involving u, v, f, c, and c', then the signs must be considered, just as in any other case, and the following interpretations of the results will be found useful :—

3 (*a*). Ratio *positive* indicates that image is *real*,
„ *negative* „ „ „ „ *virtual*,
because, if v and u are of opposite sign, then the image must be behind the mirror, and therefore virtual (Art. **34**).

3 (*b*). Ratio *positive* indicates that image is *erect*,
„ *negative* „ „ „ „ *inverted*,
because, in the first case, the object and image must be on the same side of C, and the latter is therefore *erect ;* and, in the second case, they are on opposite sides of C, and therefore the image is *inverted* (Art. **34**).

3 (*c*). Ratio *positive* indicates that image is *real*,
„ *negative* „ „ „ „ *virtual*.
Cp. 3 (*a*) and Art. **34**, I. 3 and II. 1.

When the magnification is one of the data of a problem, attention must be paid to this point. See Ex. III. 4.

EXAMPLES III.

✓ 1. An object is placed 15 cm. in front of a concave mirror of 30 cm. focal length. Find the position of the image and the ratio of its size to that of the object.

Here we have given us—

$$u = 15 \; ; f = 30.$$

Hence, substituting in—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have—

$$\frac{1}{v}+\frac{1}{15}=\frac{1}{30}. \quad \therefore \frac{1}{v}=-\frac{1}{30}.$$

$$\therefore v=-30.$$

That is, the image is 30 cm. *behind* the mirror, and is therefore *virtual*. Also, image and object are on the same side of C; therefore image is *erect*.

Also—

$$\frac{\text{Image}}{\text{Object}}=\frac{v}{u}=\frac{-30}{15}=-2.$$

That is, image is *virtual*, and twice the size of the object.

*[This problem may also be solved by the application of 2 and 3 (*c*), thus :—

From data—

$$x=-(30-15)=-15$$
$$f=30.$$

Therefore, substituting in $x\,x'=f^2$, we have—

$$-15\,x'=(30)^2;$$

or—

$$x'=\frac{30\times 30}{-15}=-60.$$

That is, the image is 60 cm. from the focus, in the same direction as the mirror, or 30 cm. *behind* the mirror.

Also—

$$\frac{\text{Image}}{\text{Object}}=\frac{f}{u-f}=\frac{30}{15-30}=-2.$$

That is, the image is *virtual*, and twice the size of the object.]

✓ 2. A pencil of rays, converging to a point 20 cm. behind a mirror, is brought to focus, by reflexion from its surface, at a point 10 cm. in front of the mirror. Determine whether the mirror is convex or concave, and find its radius of curvature.

$$\text{Here } u=-20,\ v=10,$$

$$\text{and } \frac{1}{v}+\frac{1}{u}=\frac{2}{r}.$$

$$\therefore \frac{1}{10}-\frac{1}{20}=\frac{2}{r} \qquad \therefore \frac{2}{r}=\frac{1}{20},$$

$$\text{or } r=40 \text{ and } f=20.$$

That is, the mirror is concave, and its radius of curvature is 40 cm.

3. An object, 3 cm. in length, is placed 20 cm. in front of a convex mirror of 12 cm. focal length. Find the nature and position of the image.

$$\text{Here } u = 20, f = -12$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

$$\therefore \frac{1}{v} + \frac{1}{20} = -\frac{1}{12}.$$

$$\therefore \frac{1}{v} = -\frac{1}{20} - \frac{1}{12} = -\frac{2}{15}$$

$$\therefore \frac{1}{v} = -\frac{2}{15}.$$

$$\therefore v = -7{\cdot}5.$$

That is, the image is 7·5 cm. *behind* the mirror, and is therefore *virtual.*

Also—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = -\frac{7{\cdot}5}{20}.$$

That is, image is *virtual;* and disregarding sign, we have—

$$\frac{\text{Length of image}}{3 \text{ cm.}} = \frac{3}{8}.$$

$$\therefore \text{ Length of image} = \frac{9}{8} = 1{\cdot}125 \text{ cm.}$$

*[Applying formulæ 2 and 3 (c) to this problem we get—

$$x = 20 + 12 = 32, \text{ and } f = 12.$$

$$\therefore 32\, x' = (12)^2.$$

$$\therefore x' = \frac{12 \times 12}{32} = 4{\cdot}5.$$

That is, the image is 4·5 cm. from focus in the positive direction, or 7·5 cm. *behind* the mirror.

Also—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u-f} = \frac{-12}{20-(-12)} = \frac{-12}{32}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = -\frac{3}{8}.$$

That is, the image is virtual; and disregarding sign, we have—

$$\frac{\text{Length of image}}{3 \text{ cm.}} = \frac{3}{8}.$$

$$\therefore \text{ Length of image} = \frac{9}{8} = 1{\cdot}125 \text{ cm.]}$$

4. A gas flame is placed at a distance of 8 feet from the wall of a room. Find the radius of curvature of a concave spherical mirror, and where it must be placed in order that it may produce, on the wall, an image of the gas flame magnified threefold.

Here, if x denote the distance of the mirror from the gas flame, we have—

$$u = x;\ v = x + 8.$$

And—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{x+8}{x} = 3.$$

$$\therefore\ 3x = x + 8.$$

$$\therefore\ x = 4.$$

And—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u-f}.$$

$$\therefore\ 3 = \frac{f}{4-f},\ \text{or}$$

$$12 - 3f = f.$$

$$\therefore\ f = 3 \text{ and } r = 6.$$

Or, after determining $x = 4$, we may employ 1 instead of 3 (c), thus—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore\ \frac{1}{12} + \frac{1}{4} = \frac{1}{f}.$$

$$\therefore\ \frac{1}{3} = \frac{1}{f}.$$

$$\therefore\ f = 3, \text{ and } r = 6.$$

Hence, the mirror must be placed 4 feet from the gas flame—that is, 12 feet from the wall—and its radius of curvature should be 6 feet

5. A square piece of cardboard of 1 inch side is placed at right angles to the principal axis of a concave mirror of 18 inches focal length. At what distance from the mirror must it be placed in order that an image, 9 square inches in area, may be formed?

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{f}{u-f}\right)^2.$$

$$\therefore \frac{9}{1} = \left(\frac{18}{u-18}\right)^2.$$

$$\therefore\ 3 = \pm\frac{18}{u-18}.$$

$$\therefore\ u = 24 \text{ or } 12.$$

That is, the object may be placed 24 inches in front of the mirror, or 12 inches in front of the mirror. In the former case the image is *real* and *inverted;* in the latter it is *virtual* and *erect.*

This problem may also be solved by means of formulæ 1 and 3 (*a*).

6. An object is placed 16 inches from the centre of curvature, and 12 inches from the focus of a convex mirror. Find the nature and position of the image.

Here, the distances between the focus and centre of curvature $=(16-12)=4$ inches.

$$\therefore\ r=-8 \text{ and } f=-4,$$
$$\text{and } u=16-8=8.$$
$$\therefore\ \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$
$$\frac{1}{v}+\frac{1}{8}=\frac{1}{-4}$$
$$\frac{1}{v}=-\frac{3}{8} \text{ or } v=-2\tfrac{2}{3}.$$

That is, the image is $2\frac{2}{3}$ inches behind the mirror, and is ***virtual***, *erect*, and *diminished* (Art. 34, II.).

[***Virtual*** and ***diminished*** shown by ratio—

$$\frac{v}{u}\left(=\frac{-2\frac{2}{3}}{8}=-\frac{1}{3}\right);$$

erect and *diminished* shown by ratio—

$$\frac{c}{c'}\left(=\frac{5\frac{1}{3}}{16}=\frac{1}{3}\right).]$$

7. Given a concave mirror whose focal length is 12 inches, where would you place a candle flame in order that the image of it, formed by the mirror, may be (1) real, (2) virtual.

8. A concave spherical mirror is so placed that a candle flame is situated on its principal axis at a distance of 18 inches from its surface. An inverted image, three times as long as the candle flame itself, is seen sharply defined on the wall. What is the focal length of the mirror?

9. Prove that if an object is placed at a distance of $3f$ in front of a concave mirror (of focal length f), then the image is one-half the size of the object.

10. A small object on the axis of a concave mirror, at a distance of 16 inches from it, produces a *real* image which is three times its own size. Find the focal length of the mirror.

11. A small object 0·1 inch long is placed at a distance of 3 feet from a convex mirror of 12 inches focal length. What is the length of the image and its distance from the mirror?

12. A gas flame is placed at a distance of 10 feet from the wall of a room. What must be the radius of curvature of a concave spherical mirror, and where must it be placed in order that it may produce on the wall an image of the gas flame magnified (linearly) fourfold?

13. A penny is held 8 inches in front of a convex mirror of 1 foot radius. Where will its image be, and what will be its diameter compared with that of the penny?

14. How far from a concave mirror of radius 3 feet would you place an object to give an image magnified three times? Would the image be real or virtual?

15. An object 6 cm. long is placed 1 metre in front of a concave mirror of 10 cm. focal length. Find the nature and size of the image.

16. Prove that when an object is placed midway between a concave mirror and its principal focus the image is twice as large as the object.

17. An object is held in front of a convex mirror, at a distance equal to the focal length of the mirror. Determine the size, nature, and position of the image.

18. A gas jet is placed on the principal axis of a spherical mirror 10 cm. in front of it. A real and inverted image is produced on a screen held in front of the mirror. If the length of the image is three times that of the flame, find the focal length of the mirror and the position of the screen.

19. An image produced by a convex mirror of focal length f is $1/r$th the size of the object. Show that the distance of the object from the mirror is $(r-1)f$.

20. A plane mirror is placed 6 feet in front of a concave mirror of 2 feet focal length. Find where an object must be placed between the two mirrors in order that images and object may coincide.

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

Elementary.

1. A plane mirror revolves about an axis. Explain a method of ascertaining experimentally whether or not the axis is perpendicular to the surface of the mirror.

2. Two plane mirrors are inclined at an angle of 60°: trace the path of a pencil of rays proceeding from a luminous point between the mirrors to the eye, after undergoing one reflection at the surface of each mirror.

3. State the laws of the Reflexion of Light by plane-polished surfaces, and explain fully an accurate method of proving them by experiment.

4. If a small object on the principal axis of a concave mirror is gradually moved up to the mirror from a point at a considerable distance, show what will be the simultaneous changes in the position and size of the image.

5. Apply the laws of Reflexion of Light to find the apparent position of a luminous point seen by reflexion in a plane mirror.

6. A ray of light is reflected successively by two plane mirrors, the plane of incidence being perpendicular to the line of intersection of the mirrors: prove that when the mirrors are at right angles to each other the final direction of the ray is parallel to its original direction.

7. When a ray of light falls upon a rotating mirror, show that the reflected ray turns twice as fast as the mirror.

8. Enunciate completely (in two statements) the law of Reflexion of Light. Employ it to find the positions of the images of a bright point placed between two parallel plane mirrors.

9. A candle-flame is placed at a distance of three feet from a concave mirror formed of a portion of a sphere the diameter of which is three feet. Determine the nature and position of the image of the candle-flame produced by the mirror, and state whether it is erect or inverted.

10. Sketch a concave spherical mirror exhibiting a distant luminous object, and showing the position and nature of the image of this object given by the mirror.

11. Given a concave spherical mirror, how could you find its radius of curvature by optical means alone, and without resorting to geometrical operations?

12. Rays of light from a bright gas flame pass through a small pinhole in a black screen, and are received on a sheet of ground glass. Describe by the help of a picture the image seen on the glass. What would be the effect of making the pinhole square instead of round?

13. Completely enunciate in two statements the law of Reflexion of Light, and show how to find the chief focus of a concave spherical mirror.

14. Assuming the laws of the ordinary Reflexion of Light, find the position of the image of an object placed in front of a plane mirror. What are the limits of position of the object (the mirror being supposed fixed) so that an image of it may be formed by the mirror?

15. A plane mirror, in the shape of a circle, revolves about a vertical diameter. A fixed horizontal ray of light falls upon its centre and is there reflected. Prove generally that if the mirror move through any angle the reflected ray will appear to have moved through double that angle.

16. An object 6 inches long is placed symmetrically on the axis of a convex spherical mirror, and at a distance of 12 inches from it. The image formed is found to be 2 inches long. What is the focal length of the mirror?

17. Show how to find the position of the image of an arrow placed in front of a concave spherical mirror. Explain when it is an erect, and when an inverted image.

18. Explain the formation of images by a concave cylindrical mirror. Find the relation between the distances of the two conjugate foci from the mirror. What is the position of the image of a point which is at the distance of the diameter from the reflecting surface of the cylinder?

19. A small object is placed in front of a concave spherical mirror of 6 inches radius at a distance of four inches from the surface of the mirror. Where will its image be situated? will it be erect or inverted? and what will its dimensions be compared with those of the object? Where must the object be that the image may be of the same size?

20. Explain the formation of images by means of a concave spherical mirror. How would you determine the focal length of such a mirror?

21. Explain the formation of an image by a convex mirror.

The radius of a convex mirror is 6 inches. If the linear dimensions of an object be twice those of its image, where must each be situated?

22. State the laws of Reflexion of Light. Two mirrors are placed parallel to one another, and a luminous point is placed midway between them. Show how to draw accurately the path of a ray of light which, after undergoing 3 reflections at one mirror and 4 at the other, enters an eye also placed midway between the mirrors, but at some distance from the source of light.

23. On a moonlight night when the surface of the sea is covered with small ripples, instead of an *image* of the moon being seen in the sea, a long band of light is observed on the surface of the sea extending towards the point which is vertically beneath the moon. Account for this phenomenon in accordance with the laws of reflection, illustrating your explanation by a figure.

24. A bright object is placed between two plane mirrors inclined at 45°. Draw a picture showing the path of a ray of light proceeding from the object and reaching the observer's eye after four reflections.

25. Two mirrors are inclined to each other at right angles. Show that three images of an object, placed in the angle between the mirrors, are formed, and draw the pencil of rays by which the second image can be seen by an eye looking at one mirror.

26. State the optical law on which photometric measurements are based. A gas flame and a candle are eight feet apart, the former giving out nine times as much light as the latter. Show that there are two positions in which a screen may be placed so as to be equally illuminated by the two sources, and find these positions.

27. A candle flame is placed between two vertical plane mirrors inclined to each other at an angle of 45°. Draw a figure showing the path of a ray, which, after four reflexions, enters the eye of an observer at the same level as the candle.

28. What do you understand by the intensity of illumination at a point, and how would you show that the intensity of illumination at a point, due to a given source, is inversely proportional to the square of the distance of the point from the source?

CHAPTER IV.

REFRACTION AT PLANE AND SPHERICAL SURFACES.

47. In the chapter on this subject several important relations have been established. For convenience of reference we shall here summarise the formulated expressions of these relations :—

(1) $${_b\mu_a} = \frac{1}{{_a\mu_b}}\cdot$$

That is, the index of refraction from b to a is the reciprocal of that from a to b.

(2) $${_a\mu_c} = {_a\mu_b}\ {_b\mu_c}$$

(3) $${_a\mu_b} = \frac{{_v\mu_b}}{{_v\mu_a}}$$

(4) $${_a\theta_b} = \sin^{-1}\ {_a\mu_b}.$$

The relations (3) and (4) should be learnt in words.

(5) $$D = (\phi - \phi')$$

(6) $$v = \mu u$$

(7) $$t' = \mu t$$
$$d = t\,(1 - \mu)$$

*(8) $$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$$

In formulæ which involve v and u, distances are measured from the surface of separation of the media, and the usual sign convention (Art. 37) is adopted.

In all cases μ denotes the index of refraction in the direction in which the light is travelling.

Examples IV.

1. The absolute refractive indices of diamond and glass are respectively $\frac{5}{2}$ and $\frac{3}{2}$. Find the relative indices of refraction from glass to diamond, and from diamond to glass.

Here, if ${_g\mu_d}$ denote the relative index of refraction from glass to diamond we have, from (3)—

$${_g\mu_d} = \frac{{_v\mu_d}}{{_v\mu_g}} = \frac{5}{2} \div \frac{3}{2} = \frac{5}{2} \times \frac{2}{3} = \frac{5}{3}.$$

$$\therefore\ {_g\mu_d} = \tfrac{5}{3},\text{ and by (1)}$$
$${_d\mu_g} = \tfrac{3}{5}.$$

✓ 2. Find the critical angle for water and glass, given that the index of refraction from air to glass is $\frac{3}{2}$, and that from air to water $\frac{4}{3}$.

Of the media, water and glass, glass is the denser, and by (1) and (2) we have—

$$_g\mu_w = {_g\mu_a} \cdot {_a\mu_w} = \tfrac{2}{3} \cdot \tfrac{4}{3} = \tfrac{8}{9}.$$

Now, if $_g\theta_w$ denote the critical angle for glass and water, then—

$$_g\theta_w = \sin^{-1} {_g\mu_w} = \sin^{-1} \tfrac{8}{9}.$$

That is, the critical angle for glass and water is an angle whose sine is $\frac{8}{9}$. Reference to a table of sines shows this to be 66° 44′.

2

3. A small air bubble in a piece of glass with a plane surface is 3 inches below that surface; find its apparent distance from an eye looking at it, along a normal to the surface, from a point 8 inches from the surface. (Index of refraction from air to glass $\frac{3}{2}$.)

Here, applying $t' = \mu t$ (7), and remembering that the light is supposed to be travelling from glass to air, and that therefore $\mu = \frac{2}{3}$, we have—

$$t' = \tfrac{2}{3} \times 3 = 2 \text{ inches.}$$

Therefore the apparent distance of the bubble from the eye $= 8 + 2 = 10$ inches.

✓ *4. A gold-fish globe of 6 inches radius is filled with water. Determine the apparent position of a point inside the globe, 4 inches from its surface, when seen by an eye looking along a radius of the globe.

Here, the surface at which refraction takes place is spherical; and, neglecting the action of the glass of the globe, we have—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}, \qquad (8).$$

and $\mu = \frac{3}{4}$ (water to air)
$u = 4$ inches
$r = 6$ inches
v is required.

$$\therefore \frac{3}{4v} = \frac{1}{4} - \frac{1}{24} = \frac{5}{24}.$$

$$\therefore 20v = 72.$$

$$\therefore \quad v = 3{\cdot}6 \text{ inches.}$$

That is, the apparent position of the point is inside the globe on the radius passing through its real position, and 3·6 inches from the surface.

5. A piece of plate-glass, 5 inches thick (refractive index 1·6), is placed between the eye and an object. Find what alteration will take place in the apparent distance of the object from the eye.

6. Find the relative index of refraction from Canada balsam to air. (Refer to the table of refractive indices for data.)

7. The sine of the critical angle for two media is $\frac{7}{9}$. What is the index of refraction from the rarer to the denser of the two?

8. Find the absolute refractive index of carbon disulphide, given that the relative index of refraction from carbon disulphide to glass is 0·9, and the absolute refractive index of glass is 1·512.

9. If a ray of light passes from one medium to a second, making the angle of incidence $= 45°$, and the angle of refraction equal to 30°, show that the refractive index for the media is $\sqrt{2}$.

10. The critical angle of a given medium is 60°. What is its refractive index?

Note.—When *the critical angle* or *the refractive index* of any medium is referred to, it must be understood that the other medium involved is vacuum.

11. A vessel, 6 inches deep, is filled with alcohol. What is the apparent depth of the liquid?

12. The refractive index of water is 1·33, and the velocity of light in air is 300,000,000 metres per second. Find the velocity of light in water.

13. A small air bubble in a sphere of glass, 4 inches in diameter, appears, when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be one inch from the surface. What is its true distance from the surface?

14. A small air bubble at the centre of glass sphere is seen from a point outside the sphere. What is the apparent position of the bubble? Explain.

15. A brass sphere of 2 cm. radius is surrounded by a glass shell of 6 cm. external radius. What is the apparent thickness of this shell?

CHAPTER V.

REFRACTION THROUGH PRISMS AND LENSES.

61. THE following relations, obtained in Chapter VII. of the text, may again be noticed :—

1. Prisms.

$$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2} A}. \qquad (1)\ (\text{Art. } 50.)$$

$$D = (\mu - 1) A. \qquad (2)\ (\text{Art. } 50.)$$

This formula is approximately true only when A is small, and is rigorously true only when A is infinitely small. It should, therefore, not be used in calculations except when A is small; for example, less than 10°.

2. Lenses.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right). \qquad (3)^*$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \qquad (4)\ (\text{Art. } 56.)$$

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u}, \qquad (5)$$

$$= \frac{f}{u + f}. \qquad (5a)\ (\text{Art. } 59.)$$

In the above formulæ all distances are measured from the centre of the lens, and the usual sign convention is adopted, that is, distances measured from the centre of the lens, in a direction **opposed** to the **incident light** are considered **positive**, and distances measured in the **same direction** as the **incident light** are considered **negative**. In accordance with this convention the **focal length** (f) of a **convex lens** will be **negative**, and that of a **concave lens positive**.

In applying the formulæ the rules given in Art. **37** must be attended to. Of these (1) and (2) are so important, and their neglect so often leads to mistakes, that we shall again deal with them in their relation to the formulæ here considered.

(1) On substituting in any formula a numerical value for any of the symbols, the sign of the former must always be attached. For example, take the formula—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

If the image of an object, placed 20 cm. from a lens, be formed at a point 40 cm. *on the other side of the lens*, then, to find f we have—

$$u = 20, v = -40;$$

and

$$\therefore \frac{1}{-40} - \frac{1}{20} = \frac{1}{f};$$

$$\therefore -\frac{3}{40} = \frac{1}{f};$$

and

$$f = -\frac{40}{3} = -13\tfrac{1}{3} \text{ cm};$$

that is, the lens is **convex**, and its focal length is $13\frac{1}{3}$ cm.

(2) In applying a formula to determine one of the involved distances, *no sign must be given to the unknown distance.* Thus, in the example given above, no sign is at first given to f, but the result, when worked out, shows it to be negative.

In applying formulæ (5) and (5a), which express the relative size of image and object, the question of sign should be carefully attended to, for the interpretation of the result is simple and important. In these formulæ, a *positive* result indicates that the image is *virtual* and *erect;* for the image and object are then on the same side of the lens. Similarly, a *negative* result indicates that the image is *real* and *inverted*, the image and object being then on opposite sides of the lens. (See footnote, Art. **58**.)

EXAMPLES V.

1. The refracting angle of a prism is 60°, and the minimum deviation produced in a pencil of monochromatic light is 40°. Find the refractive index of the prism, given that $\sin 50° = \cdot766$.

Here, applying—

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A},$$

we get—

$$\mu = \frac{\sin \frac{1}{2}(60 + 40)}{\sin \frac{1}{2}(60)} = \frac{\sin 50}{\sin 30}.$$

$$\therefore \mu = \frac{\cdot766}{\frac{1}{2}} = 1\cdot532.$$

✓ *2. Find the focal length of a double concave lens, the radii of curvature of its faces being respectively 25 cm. and 50 cm., and the refractive index of its material being 1·5.

Here, in formula (3)—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

we have, supposing the light to be incident on the more concave face—

$$\mu = 1{\cdot}5,\ r = 25 \text{ cm.},\ s = -50 \text{ cm.}$$

$$\therefore\ \frac{1}{f} = (1{\cdot}5 - 1)\left(\frac{1}{25} + \frac{1}{50}\right)$$

$$= \frac{1}{2} \times \frac{3}{50} = \frac{3}{100}.$$

$$\therefore\ f = 33\tfrac{1}{3} \text{ cm.}$$

If we suppose the light to be incident on the other face we get the same result; thus, as before—

$$\mu = 1{\cdot}5,\ r = 50 \text{ cm.},\ s = -25 \text{ cm.}$$

$$\therefore\ \frac{1}{f} = (1{\cdot}5 - 1)\left(\frac{1}{50} + \frac{1}{25}\right).$$

$$\therefore\ \frac{1}{f} = \frac{1}{2} \times \frac{3}{50} = \frac{3}{100}, \text{ and } f = 33\tfrac{1}{3} \text{ cm.}$$

✓ 3. An object is placed 12 inches from a convex lens of 8 inches focal length. Find the position and nature of the image.

Here, in formula (4),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have—

$$u = 12 \text{ inches}, f = -8 \text{ inches (\textit{convex} lens)},$$

and v is required—

$$\therefore\ \frac{1}{v} - \frac{1}{12} = \frac{1}{-8}.$$

$$\therefore\ \frac{1}{v} = -\frac{1}{8} + \frac{1}{12} = -\frac{1}{24}.$$

$$\therefore\ v = -24 \text{ inches};$$

that is, the image is 24 inches on the other side of the lens.

Again, applying (5), we have—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{-24}{12} = -2;$$

that is, the image is twice the size of the object, and is *real* and *inverted.*

2

4. An object, 3 cm. long, is placed 10 cm. from a concave lens of 20 cm. focal length. Find the size and nature of the image. Here, from (5a) we get—

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u+f} = \frac{20}{10+20} = \frac{2}{3}.$$

$$\therefore \frac{\text{Length of image}}{3 \text{ cm.}} = \frac{2}{3}.$$

$\therefore$ Length of image = 2 cm., and the image is *virtual* and *erect*.

A more usual, but less direct method of working this question is, first to determine v, and then to determine the size and nature of the image from formula (5).

5. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius at a distance of 6 inches from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed?

[In problems such as this, where by reflexion and refraction the image of the object is made to coincide with the object itself, the solution is easy if we remember that rays diverging from a point in the object, *on the principal axis*, return to the same point, and therefore travel to and fro by the same paths. But, if a ray, after reflexion at a mirror, return along its incident path, it follows that it must be travelling *along a normal to the mirror*.]

> In this case we know that, after the first refraction through the lens, the rays of the refracted pencil—originally diverging from a point in the object on the principal axis—are normal to the mirror, and therefore diverge from its centre of curvature. To find the position of the object we have therefore only to find a point on the principal axis such, that rays diverging from this point appear, after refraction through the lens, to diverge from the centre of curvature of the mirror.

Hence, in the formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have—

$v = 6$ inches, $f = 12$ inches, and u is unknown.

$$\therefore \frac{1}{6} - \frac{1}{u} = \frac{1}{12}.$$

$$\therefore \frac{1}{u} = \frac{1}{6} - \frac{1}{12} = \frac{1}{12}.$$

$$\therefore u = 12 \text{ inches.}$$

That is, the object must be placed 12 inches from the lens on the side remote from the mirror.

6. Find the focal length of a single lens that is optically equivalent to two thin lenses in contact, and of focal lengths f_1 and f_2 respectively.

> When a luminous point is placed on the principal axis of a convex lens (*A*) and at a distance *a* from it, an image is formed 10 inches from the lens on the other side. If a second lens (*B*) is placed close to *A*, the image is 15 inches off. Determine the focal length of the lens *B*, and state whether it is concave or convex.

Imagine light from a point P, at a distance u from the centre of the combination,* to be incident first on the lens of focal length f_1. Then, *considering the action of this lens only*, the focus of the refracted pencil will be at a point, P′, at a distance v' from the lens, such that we have—

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1}. \qquad (1)$$

But this refracted pencil passes through the second lens, and after doing so is refracted through another point P″, at a distance v from the lens, such that—

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2}. \qquad (2)$$

The combined action of the lenses is thus to cause a pencil diverging from P, at a distance u from the centre, to be refracted through P″, at a distance v from the centre. Therefore, if F be the focal length of the combination, we have—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{\mathrm{F}}. \qquad (3)$$

But, by adding (1) and (2) we get—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}. \qquad (4)$$

Therefore, from (3) and (4) we get—

$$\frac{1}{\mathrm{F}} = \frac{1}{f_1} + \frac{1}{f_2}.$$

$$\therefore\ \mathrm{F} = \frac{f_1 f_2}{f_1 + f_2};$$

that is, a single lens of focal length, $\frac{f_1 f_2}{f_1 + f_2}$, is optically equivalent to two thin lenses in contact, and of focal lengths f_1 and f_2 respectively.

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* The thickness of the lenses is supposed to be so small, compared with the other distances involved, that the centre of the combination may be taken at any point in their combined thickness.

In the second part of the question, the action of the lens B is evidently to cause a pencil of rays, originally converging to a point P, 10 inches behind the lenses to become less convergent, and to converge to a point P′, 15 inches behind the lenses. Thus, P and P′ are conjugate foci with respect to the lens B, P′ being the image of P, and, therefore, in the formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have—

$$u = -10 \text{ inches}, \ v = -15 \text{ inches, and } f \text{ is unknown.}$$

$$\therefore -\frac{1}{15} - \frac{1}{-10} = \frac{1}{f}.$$

$$i.e., -\frac{1}{15} + \frac{1}{10} = \frac{1}{f}.$$

$$\therefore \frac{1}{30} = \frac{1}{f}.$$

$$\therefore f = 30;$$

that is, the lens is *concave*, and its focal length is 30 inches.

7. Show that if the angle of a prism be greater than twice the critical angle for the medium of which it is composed, no ray can pass through it.

8. The angle of a prism is 60°, and the refractive index of its material $\sqrt{2}$. Show that the minimum deviation is 30°.

9. A glass prism of refracting angle 5° is immersed in water; find the approximate deviation produced in a ray of light for which the absolute refractive indices of glass and water are respectively $\frac{3}{2}$ and $\frac{4}{3}$.

10. The minimum deviation produced by a hollow prism, filled with a certain liquid, is 30°; if the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

11. Show that when a ray of light is refracted through a prism, in the position of minimum deviation, the course of the ray in the prism is perpendicular to the line bisecting the angle of the prism.

*12. In order to determine the refractive index of a double convex lens, the radii of curvature of its surfaces were measured and found to be 30 cm. and 31 cm. respectively. Its focal length was also determined, and found to be 30·5 cm. Find the refractive index of the glass.

*13. Find the focal length of a plano-convex lens, given that the radius of curvature of its convex surface is 50 cm., and that the refractive index of its material is 1·6.

*14. Prove that the focal length of a plano-concave glass lens is equal to twice the radius of the concave surface. ($\mu = \frac{3}{2}$.)

15. A gas flame is at a distance of 6 ft. from a wall. Where must a convex lens, of 1 ft. focal length, be placed in order to give a distinct image of the flame on the wall? Explain your result.

16. An object, 1 inch long, is placed at a distance of 1 ft. from a convex lens of 10 inches focal length; find the nature and size of the image.

17. If an object, 10 cm. from a convex lens, has its image magnified 4 times, what is the focal length of the lens?

18. An object is at a distance of 3 inches from a convex lens of 10 inches focal length. Find the nature and position of the image.

19. An object is placed 6 inches from a lens, and an image, 3 times as large, is seen on the same side of the lens as the object. Find the focal length of the lens.

20. A convex lens of 10 inches focal length is combined with a concave lens of 6 inches focal length. Find the focal length of the combination.

21. Find the focal length of a lens which is equivalent to two thin convex lenses of focal lengths 20 cm. and 30 cm. placed in contact.

22. A convex lens of focal length 12 cm. is placed in contact with a concave lens, and the focal length of the combination is found to be 24 cm. Calculate the focal length of the concave lens.

23. A convex lens of 6 inches focal length is used to read the graduations of a scale, and is placed so as to magnify them 3 times; show how to find at what distance from the scale it is held, the eye being close up to the lens.

24. The image formed by a convex lens is n times the size of the object. Show that the distance of the object from the lens is $\frac{n+1}{n}f$.

25. A candle flame is placed 6 inches from a plane mirror, and a convex lens, of 3 inches focal length, is placed between the candle and the mirror, and 2 inches from the latter. Find the position of the image.

26. A candle flame is placed 20 cm. from a plane mirror. Find where a convex lens of 5 cm. focal length must be placed in order that the image of the flame may coincide with the flame itself.

27. The focal length of a lens *in vacuo* is 2 feet. The refractive indices of glass and water being respectively $\frac{3}{2}$ and $\frac{4}{3}$, find the focal length of the lens when placed in water.

28. Show (by application of formula 5a above) that the image formed by a concave lens is always less than the object.

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

Elementary.

1. Explain, in non-mathematical language, what is meant by the "Index of Refraction" of a transparent medium.

Also describe the phenomenon known as "total reflexion," and show how its occurrence in a given medium is connected with the index of refraction of that medium.

2. State the laws of the Refraction of Light by such substances as Water or Glass, and describe and explain experiments by which they can be demonstrated.

3. If a candle is placed at a distance of 6 feet from a wall, and a distinct image of the flame is produced upon the wall by a lens held at 1 foot from the candle, show that a distinct image will also be produced when the lens is at 5 feet from the candle, and compare the sizes of the two images.

4. A beam of light, on passing obliquely from air into water, is bent away from the surface of the water; and a straight stick, with one end immersed obliquely in water, appears to be bent towards the surface of the water. Show that these are illustrations of the same law of Refraction.

5. Explain how to draw a figure to represent the formation of a real magnified image of a small object by a lens. If a real image five times as high as the object is to be thrown on a screen at a distance of 36 inches from the object, show what must be the focal length of the lens employed.

6. A beam of light issues from a given bright point 3 feet above the surface of still water, and falling obliquely on the surface, is divided into two parts, one of which is reflected and the other refracted. Find the position of the point of incidence and its distance from the bright point, so that the reflected and refracted beams may be at right angles to each other.

[The index of refraction from air to water is $\frac{4}{3}$.]

7. An object is moved from a considerable distance on the principal axis of a convex lens up to the lens. Find the corresponding changes in the position and size of the image.

8. A bright point, 6 inches above the surface of still water, is reflected from the bottom of the vessel, which is 2 feet deep as well as from the surface of the water. Show how to find the positions of the images formed by the reflections. ($\mu = \frac{4}{3}$.)

9. Enunciate by aid of a sketch, and in two statements, the law of Refraction when light passes from a rarer to a denser medium. Also point out what is meant by the Critical Angle.

10. A simple lens is used as a magnifier. Sketch the relative positions of the object (an arrow) and its image.

The same lens is used as in photography. Sketch the relative positions of the object and its image.

11. Enunciate completely, by aid of a sketch, the law of Refraction. State what you mean by the Index of Refraction, and what by the Critical Angle.—The index of refraction from air to water is $\frac{4}{3}$: what is the sine of the critical angle in this case?

12. Prove that the apparent depth of a luminous object beneath a surface of water is only $\frac{3}{4}$ths of its real depth.

[The index of refraction for water is $\frac{4}{3}$.]

13. When an object is to be photographed, an image of it is first obtained on a ground-glass screen, by means of the lens of the camera. Describe the nature and position of this image, and explain its formation.

14. Explain "Angle of Reflexion," "Angle of Refraction," "Critical Angle." Is there any displacement of an object which is seen through a sheet of plate glass? Give reasons, aided by a diagram, for your reply.

15. Given the focal length of a convex lens, explain generally how it is possible to find the size of the image of the sun which such a lens will give. In what respect will this image be altered by diminishing the area of the lens without altering its curvature?

16. A ray of light passes from air into glass, the refractive index of glass with regard to air being 1·5. Given the angle of incidence at the common surface, draw a diagram to show how the angle of refraction may be accurately determined.

17. The chief focal length of a lens is 12 inches; how far must I place a luminous object from the lens in order to obtain an image twice as large every way as the object?

18. What is meant by a "refractive index" of a substance? Explain the fact that an aquarium tank appears to be much shallower (from front to back) than it really is; and point out in what way the difference between the apparent and true thickness is connected with the refractive index of the water in the tank.

19. Describe the two main categories under which lenses may be classified. A lens of water is enclosed in a rectangular envelope of glass. What kind of lens will the combination form?

20. If the refractive index of a ray of light in passing from air to water be $\frac{4}{3}$, and in passing from air to glass $\frac{3}{2}$, find, by aid of a diagram, what it will be for the ray when passing from water to glass.

21. Light proceeds from a point at the bottom of a lake. Make a careful drawing of the pencil of rays after emergence from the water, and find the geometrical focus of the pencil.

22. What is the critical angle of a transparent medium? Describe what a fish would see on looking towards the surface of the water in directions differently inclined to the horizon, and illustrate your description by a diagram.

23. What is the focal length of a lens? A circle an inch in diameter, a convex lens whose focal length is 6 inches, and a second lens whose focal length is 10 inches are placed so as to have a common axis. The distance from the circle to the first lens is 10 inches, and from the first lens to the second 36 inches. What images of the circle will be formed, where will they be situated, and what will be their dimensions?

24. How would you experimentally verify the laws of Refraction? What condition is necessary in order that a ray of light may be able to emerge from the plane surface of a refracting medium?

25. What is the index of refraction of a transparent medium? What is the position of *minimum deviation* for a prism? Describe and explain the appearance presented when the image of a window is looked at through a prism with its edge vertical.

26. Given the focal length of a lens, show how, by a geometrical construction, to find the position and magnitude of the image of an object whose distance from the lens is given.

An object whose length is 2 inches is placed 6 inches in front of a convex lens whose focal length is 4 inches. What is the length of the image?

27. Distinguish between a real image and a virtual image. Explain the action of a convex lens when used as a magnifying glass. Is the image seen by the eye real or virtual?

28. Show how to find the position and size of the virtual image of a given object, formed by a concave lens of known focal length. A concave lens whose focal length is 12 inches is placed on the axis of a concave mirror of 12 inches radius, at a distance of 6 inches

from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object itself. Where must the object be placed?

29. What is meant by the statement that the index of refraction of water is $\frac{4}{3}$? *Walking* by the side of a shallow stream of clear water of uniform depth, the gravelled bottom appeared to possess a wave-motion, the trough of the wave being always vertically beneath the observer. Explain this by means of a diagram.

30. What is meant by the statement that the refractive index of water is 1·333? How is the critical angle for water found? An object is fixed one foot above the surface of still water; show how to find the apparent position of this object, as seen by an eye two feet vertically under it.

31. What is meant by saying that the refractive index of water with respect to air is $\frac{4}{3}$?

If the refractive index of water with respect to oil of turpentine be $\frac{9}{10}$, show how to find the refractive index of oil of turpentine with respect to air.

32. What is meant by the refractive index of a substance, and by total internal reflexion? Describe some experiment by which the phenomenon of total internal reflexion may be produced and observed. State also how the minimum angle of incidence at which total internal reflexion takes place may be determined.

33. An object 3 inches in height is placed at a distance of 6 feet from a lens, and a real image is formed at a distance of 3 feet from the lens. The object is then placed 1 foot from the lens. Where, and of what height will the image be?

EXAMINATION QUESTIONS.

QUESTIONS SET AT VARIOUS UNIVERSITY EXAMINATIONS.

Elementary.

1. What is meant by the Focal Length of a Convex Lens? Show how to find it (1) by aid of the sun, (2) by an artificial flame.

2. An arrow, pointing towards the observer, is seen by internal reflection in an isosceles right-angled prism. Explain the difference in, and give a sketch of, the images seen, according as the prism is three-sided, or a four-sided Wollaston prism.

3. What is the focal length of a lens, and how would you determine it experimentally? In the case of a convex lens, if the object be as near as possible to the image, where must the lens be?

4. Show by a drawing how you would employ a right-angled isosceles glass prism to bend a beam of light at right angles. Will any light be lost at the hypotenuse? State fully the reasons for your answer.

Explain the formation of an image by a convex mirror.

5. How would you determine the focal length of a convex lens if sunlight were not available?

6. What is the centre of a lens? Under what circumstances is the centre of a lens midway between its surfaces? Two equal lenses are placed side by side in the same plane, with their centres 3 inches apart. Two objects of the same size and shape, but of different colours, are placed behind the lenses at a distance of twice its focal length from each lens, and with their centres 6 inches apart, the line joining the centres of the objects being parallel to that joining the centres of the lenses. How will the images be situated, and what will be seen by an observer situated at a considerable distance in front of the lenses?

7. What is the focal length of a lens? How may the focal length of a concave lens be determined?

8. A candle is placed at a fixed distance opposite a wall. A convex lens, held between the candle and the wall, throws on the wall a well-defined magnified image of the candle flame when it is 1 foot from the candle, and a well-defined diminished image when it is 11 feet from the candle. Find the focal length of the lens.

9. How is the focal length of a convex lens best determined without the aid of sunlight?

An object is placed 8 inches from the centre of a convex lens, and its image is found 24 inches from the centre on the other side of the lens. If the object were placed 4 inches from the centre of the lens, where would the image be?

Advanced.

10. A small object is placed close to a thick plate-glass mirror. An eye near the mirror observes not one image of the object only, but a great number. Explain the formation of these. Which of them is the brightest, and why?

11. A small gas flame is placed on the axis of a lens distant from it 120 cm. By means of a ground-glass screen an inverted image of the gas flame is found on the farther side of the lens 200 cm from it. What is the nature of the lens, and what is its focal length?

12. Describe the method by which Fizeau investigated the Velocity of Light.

13. Explain the method of obtaining a pure spectrum.

14. Explain how to determine the focal length of a convex lens. If an object at a distance of 3 inches from a lens has its image magnified three times, find the focal length of the lens.

15. What is meant by the dispersive power of a prism? Under what conditions is it possible by means of prisms to obtain deviation of a beam of white light without dispersion? Hence explain the construction of an achromatic object glass.

16. Describe Foucault's method of measuring the velocity of light by means of a rotating mirror. What is the effect of introducing a tube of water (with glass ends) between the rotating and fixed mirrors, and what relation is there between the velocity of light in a medium and its refractive index?

17. Explain the formation of the prismatic spectrum. If a horizontal beam of sunlight is admitted into a dark room through a narrow vertical slit, what arrangement of apparatus is required to throw a sharply defined spectrum upon a screen? Give a diagram.

SOUND.

CHAPTER I.

THE VELOCITY OF SOUND.

On theoretical considerations the velocity of sound is proved to be given by

$$V = \sqrt{\frac{\text{elasticity}}{\text{density}}}.$$

In a rod $$\mathrm{V} = \sqrt{\frac{\text{Young's modulus}}{\text{density}}}.$$

In a gas $$\mathrm{V} = \sqrt{\frac{\text{adiabatic elasticity}}{\text{density}}} = \sqrt{\frac{\text{K. pressure}}{\text{density}}},$$

where K = **ratio of the specific heats** at constant pressure and volume, and for air = 1·404.

Since the density of a gas is proportional to its pressure, V is **independent of the pressure.** Now the ratio $\frac{\text{pressure}}{\text{density}}$ for any given mass of gas is proportional to the absolute temperature.

$$\therefore \mathrm{V} \propto \sqrt{\mathrm{T}}, \text{ consequently } \mathrm{V}_t/\mathrm{V}_o = \sqrt{\mathrm{T}_t/\mathrm{T}_o},$$

where V_t, V_o are velocities and T_t, T_o absolute temperatures at temperature $t°$, and 0° respectively on the Centigrade scale. From the above

$$\mathrm{V}_t = \mathrm{V}_o\sqrt{1 + \frac{t}{273}} \equiv \mathrm{V}_o\left(1 + \frac{t}{546}\right)$$

$$\equiv \mathrm{V}_o(1 + \cdot 00184\,t) \equiv \mathrm{V}_o + 61\,t,$$

for V_o in air = 33210 cms. per second.

In **English units**, $V_o = 1090$ feet per second, and the equation becomes $V_t = 1090 + 2t$.

From the first equation we see that the velocities of sound in different gases are inversely as the square roots of their densities, *i.e.*

$$\frac{V_1}{V_2} = \sqrt{\frac{D_2}{D_1}}.$$

In free space the **intensity** of sound varies inversely as the square of the distance from the sounding body.

EXAMPLES I.

1. If the velocity of sound in air at 0°C. is 33240 cms., what is its value at 50° and 100°C.?

$$V_t = V_o\sqrt{\frac{273+t}{273}}.$$

When $t = 50$, $V_t = V_o\sqrt{\frac{323}{273}} = 36170.$

When $t = 100$, $V_t = V_o\sqrt{\frac{373}{273}} = 38860.$

From these results we see that when t is large we must not use the approximate formula.

2. Find the velocity of sound in air at 0°C. and 760 mm. pressure.

Given Density of mercury = 13·596 gms./cm.^{3}

$g = 981{\cdot}17$ cms./sec.^{2}

Mass of 1 c.c. of air = ·001293 gms.

Pressure = force per sq. cm.

Suppose barometer tube has a cross section of 1 sq. cm., then—

Volume of mercury = 76 c.cs.

Mass ,, ,, = 76 × 13·596 gms.

Weight = force = 76 × 13·596 × 981·17,

$$\therefore v = \sqrt{\frac{1{\cdot}404 \times 76 \times 13{\cdot}596 \times 981{\cdot}17}{{\cdot}001293}}$$

$$= 33180 \text{ cm. per sec.}$$

3. In the well known experiment for finding the velocity of sound in water at the Lake of Geneva the interval of time between flash and sound was 9·75 seconds. The distance being 14 kilometres, find the velocity of sound in water.

$$\text{Velocity} = \frac{\text{space}}{\text{time}}$$

$$= \frac{1400000 \text{ cms.}}{9{\cdot}756 \text{ seconds}} = 143600 \text{ cms. per sec.}$$

4. Find the velocity of sound in hydrogen at 0° assuming velocity in air at 0° = 33240 and that air is composed of nitrogen and oxygen in the volumetric ratios of 4 to 1.

$$\text{Mass of 1 c.c. hydrogen} = x$$

$$\text{,, ,, 1 ,, air} = \tfrac{4}{5} \times 14x + \tfrac{1}{5} \times 16x$$

$$= 11{\cdot}2x + 3{\cdot}2x$$

$$= 14{\cdot}4x$$

$$\frac{\text{Velocity in hydrogen}}{\text{Velocity in air}} = \sqrt{\frac{\text{density of air}}{\text{density of hydrogen}}}.$$

$$\therefore V_H = 33240 \times \sqrt{14{\cdot}4}$$

$$= 126{,}100 \text{ cms. per second.}$$

5. A pistol was fired at one end of a cast iron tube 1·61 kilometres long. At the other end two sounds were heard with an interval of 4·40 seconds between them. If velocity of sound in air at the time was 34000 cms. per sec., find the velocity in cast iron.

6. Calculate the distance of a thundercloud if 3 seconds elapse between the lightning flash and the hearing of the thunder, the temperature being 15°.

Vel. at 0° = 1090′ per sec.
Temp. coeff. = 2′ per degree.

7. Find the velocity of sound in a steel wire.

$$\text{Young's modulus} = 2 \times 10^{12} \text{ dynes per sq. cm.}$$

$$\text{Density} = 8.$$

8. A metronome beating half seconds was carried along a long hall until it was noticed that the echo from one beat coincided with the next beat. Find the distance between the metronome and the end of the wall.

$$\text{Vel.} = 340 \frac{\text{metre}}{\text{sec.}}.$$

9. A cannon is placed 550 yards from a long perpendicular line of smooth cliffs. An observer at the same distance from the cliffs hears the cannon fire 4 seconds after he sees the flash.

If the velocity of sound is 1100′ per sec., when will he hear the echo from the cliff? (S. and A., 1894.)

10. On one occasion when air was at the freezing point of water a sound made at a given point was heard at a second point after an interval of 10 seconds. Find the temperature of the air on a second occasion when the time taken to travel between the two points was 9·562 seconds. (S. and A., 1905.)

11. The sound of a cannon, which is known to be 2½ miles distant, is heard 12 seconds after the flash is seen. Find the velocity of sound at the time of the observation. (1897.)

12. A sound is produced between two parallel obstacles, from each of which several repetitions of an echo are heard. If the first and second echoes are heard three-quarters and five-quarters of a second respectively after the sound was produced, when will the third echo be heard? If the velocity of sound is 1100 feet per second, what is the distance between the obstacles? (1899.)

13. A ship three miles from the shore fires a gun. The flash is seen and after 14 seconds the report is heard. What is the velocity of sound and what is approximately the temperature, supposing the velocity at 0°C. to be 1090 feet per second? (1901.)

14. Find Young's modulus for a copper rod whose density is 8·8, which transmits sound waves with a velocity of 3963 metres per second.

15. An echo repeated five syllables when the temperature was 20°C. At what distance from the observer was the reflecting surface, taking the rate of speaking at five syllables per second?

16. An echoing cliff 450 feet distant repeats four syllables. Taking the rate of speaking at five syllables a second, find the temperature of the intervening air.

CHAPTER II.

FREQUENCY AND MUSICAL SCALES.

1. $$V = n\lambda,$$

V = velocity of sound.

n = frequency of note = number of vibrations per second.

λ = wave-length of note.

2. **Doppler's Principle.**—Alteration of frequency with motion of source, observer, and medium,

$$n' = n\frac{V + w - u_o}{V + w - u_s},$$

n = frequency of note emitted by source,

n' = ,, ,, ,, as heard by observer,

$$\left.\begin{array}{rl} w = & \text{velocity of wind} \\ u_o, u_s = & \text{velocities of observer and source} \end{array}\right\} \begin{array}{l} \text{these are positive when} \\ \text{measured in same direction.} \end{array}$$

If the motions of the wind, observer, and source are not parallel to the line joining observer and source, we must take their components along that line.

3. If two notes of frequencies m and w are sounded together the frequency of the **beats** is $m - w$.

The Determination of Frequency.

1. **The Siren.** If the disc makes N revolutions in t secs., and there are m holes in it, the frequency of the note is equal to $\frac{Nm}{t}$.

2. **Savart's Toothed Wheel.** If there are m teeth on the wheel and the wheel makes N revolutions in t secs., the frequency of the note given out is $n = Nm/t$.

Interval. The interval between two notes of frequencies m and n is equal to m/n.

Musical Scales.—(1) **The Diatonic Scale.**

Notes—	C,	D,	E,	F,	G,	A,	B,	C
Relative Frequencies—	1,	9/8,	5/4,	4/3,	3/2,	5/3,	15/8,	2
or	1,	1·125,	1·25,	1·333,	1·5,	1·667,	1·875,	2.

Intervals.—Octave = 2/1 (= C′/C). Major Third = 5/4 (= E/C). Fourth = 4/3 (= F/C). Fifth = 3/2 (= G/C). Minor Third = 6/5 (= C/A).

Major Tones	D/C,	G/F,	B/A =	9/8,
Minor Tones	E/D,	A/C,	=	10/9,
Major Semitones	F/E,	C′/B,	=	16/15.

(2) **The Tempered Scale.**

Notes—	C,	D,	E,	F,	G,	A,	B,	C
Relative Frequencies—	1,	$2^{2/12}$,	$2^{4/12}$,	$2^{5/12}$,	$2^{7/12}$,	$2^{9/12}$,	$2^{11/12}$,	2
or	1,	1·120,	1·260,	1·335,	1·498,	1·682,	1·888,	2

which is a remarkable approximation to the Diatonic Scale.

EXAMPLES II.

1. Taking $V = 33{\cdot}240$ cms. per second, the frequency of a C′ fork as 512, find the wave-length of the note in air.

$$33{\cdot}240 = 512\lambda,$$
$$\therefore\ \lambda = 64{\cdot}9 \text{ cms.}$$

2. A railway train travelling due N. at the rate of 3000 cms. per sec. has a whistle which emits a note of frequency 600. A N. wind is blowing at the rate of 2000 cms. per second. Find the frequency of the note as heard by an observer (*a*) to N. of train, (*b*) to S. of train, (*c*) due E. of train.

$$V = 34000 \text{ cms. per second.}$$

(*a*)
$$w = -2000,\quad u_o = 0,\quad w_s = 3000.$$
$$n' = 600 \times \frac{34000 - 2000}{34000 - 2000 - 3000}$$
$$= 600 \times \frac{32}{29}$$
$$= 662.$$

(*b*) $w = 2000. \qquad u_s = -3000.$

$$n' = 600 \frac{34000 + 2000}{34000 + 2000 + 3000}$$

$$= 600 \times \frac{36}{39}$$

$$= 554.$$

(*c*) Here the wind is blowing at right angles to the line joining the source and the observer, the component of w along this line is zero, also the motion of the source is perpendicular to this line, therefore similarly the component of u_s along this line is zero, and consequently

$$n' = n = 600.$$

3. The disc of a siren has 30 holes. A note of frequency 455 is required. How many revolutions must it make in 10 secs.?

$$455 = \frac{\text{N} \times 30}{10},$$

$$\therefore \text{N} = 151\tfrac{2}{3}.$$

4. A Savart's wheel has two sets of teeth on its circumference, containing 40 and 42 teeth respectively. If the wheel makes 15 revolutions per second find (1) the frequencies of the two notes, (2) the frequency of the beats, (3) the interval between the two notes.

(1) Frequency due to one set $= 15 \times 40 = 600.$

Frequency due to other $= 15 \times 42 = 630.$

Frequency of beats $30.$

Interval $= \frac{630}{600} = \frac{42}{40} = 1{\cdot}05.$

5. What is the interval between B and B flat on the tempered scale?

Taking frequency of C as 1, $\text{B} = 2^{\frac{11}{12}}.$

$$\text{B flat} = 2^{\frac{10}{12}},$$

$$\therefore \text{interval} = 2^{\frac{11}{12}} / 2^{\frac{10}{12}} = 2^{\frac{1}{12}}$$

$$= 1{\cdot}059.$$

6. At what speed must two trains each be travelling in opposite directions that the pitch of the whistle of one as heard from the other may change by a whole tone, *i.e.* in the ratio of 9 : 8 when they pass?

$$\text{V} = 1100 \text{ feet per sec.}$$

7. If the frequency of a tuning-fork is 128, and the number of vibrations per hour of a second fork exceeds that of the first by 300, how many beats will there be in a minute if the two are sounded together? (1898.)

8. What is meant by the wave-length of an undulation? A pendulum which swings from rest to rest through a distance of six inches has a pencil attached to the bob which traces a curve on a piece of paper placed behind the bob and parallel to the plane of swing. The paper falls with a uniform velocity of 1 foot per second. Give a sketch of the curve traced on the paper and find its wave-length. (1902.)

9. A certain fork has a frequency of 300. Find the length of the waves it sets up (*a*) in air at 0°, (*b*) in air at 100°.

10. A fork having a period of 1/300 seconds sets up waves 3 feet long in carbon dioxide. Find the velocity of sound in that gas.

11. The velocity of sound in air being taken as 1120 feet per second, how many vibrations must a fork of frequency 264 make before it is heard 154 feet away?

12. A wheel with 33 teeth touches a card as it rotates, and yields a note with a frequency of 1023. How many revolutions per second is the wheel making?

13. A disc pierced with 24 holes makes 100 revolutions in 5·2 seconds, and when a jet of air is blown against the row of holes yields a note in unison with that of a certain fork. What is the frequency of the fork?

14. In a siren, having a disc with 30 holes, 900 revolutions were made in one minute. Find the frequency.

15. A cogwheel containing 64 cogs revolves 210 times per minute. What is the frequency of the musical note produced when a card is held against the revolving teeth? Find also the wave-length corresponding to the note if the velocity of sound is 1126·4 feet per second. (S. and A. 1896.)

16. Two vibrating forks make simultaneously parallel wave-traces on a moving plate. Two lines are drawn across these traces perpendicular to them at a certain distance apart. On counting the waves it is found that one fork makes 18 waves while the other makes 24. If the former fork be a G, what is the latter?

CHAPTER III.

TRANSVERSE VIBRATIONS OF STRETCHED STRINGS OR WIRES.

The number of vibrations per second made by a stretched string depends upon its **length** l, its **mass** m, and the **stretching force** F, and is given by the formula

$$N = \tfrac{1}{2}\sqrt{\frac{F}{lm}}.$$

The mass m of the string is equal to $\pi r^2 l\delta$, where r is the radius and δ the density.

Hence
$$N = \frac{1}{2rl}\sqrt{\frac{F}{\pi\delta}}.$$

Therefore

(1) The frequency of vibration is inversely as the length with given stretching force, i.e. $n \propto \frac{1}{l}$.

(2) With constant length the frequency is proportional to the square root of the stretching force, i.e. $n \propto \sqrt{F}$.

(3) The stretching force and the length being the same for two strings, their frequencies are inversely as the square roots of the mass, and are independent otherwise of the material, i.e. $n \propto \sqrt{\frac{1}{m}}$.

The **velocity of propagation** of a disturbance or a wave along a stretched string is given by

$$V = \sqrt{\frac{F}{m}},$$

where F = stretching force,
m = mass per unit length.

EXAMPLES III.

1. A rope weighing half a pound to the foot is stretched horizontally by a force equal to the weight of 100 lbs. If the rope is shaken at one end, with what velocity do the waves run along it?

$$\text{Stretching force} = 100 \times g \text{ in absolute units}$$
$$= 100 \times 32.$$
$$m = \tfrac{1}{2}\text{ lb.},$$
$$\therefore V = \sqrt{\frac{100 \times 32}{\frac{1}{2}}} = \sqrt{100 \times 64} = 10 \times 8 = 80 \text{ feet per sec.}$$

2. A wire a metre long has a mass of 10 grammes. How many kilogrammes must be hanged on to it to make it yield a note of frequency 300?

$$N = \tfrac{1}{2}\sqrt{\frac{F}{lm}},$$
$$\therefore 300 = \tfrac{1}{2}\sqrt{\frac{F}{100 \times 10}},$$
$$\therefore 90000 \times 4 \times 100 \times 10 = F,$$
$$\therefore F = 360000000 \text{ dynes}$$
$$= \frac{360000000}{981 \cdot 2} \text{ grammes}$$
$$= \frac{360000}{981 \cdot 2} \text{ kilogrammes}$$
$$= 366 \cdot 8 \text{ kilos.}$$

3. A piece of steel wire 30 cms. long and ·62 mm. in diameter when stretched by 10 kilos. gives the same note as a brass wire 25 cms. long and ·5 mm. diameter stretched by an unknown weight. Find the latter in kilos. Density of steel 7·9, of brass 8·4 gms. per c.c.

$$N = \frac{1}{2rl}\sqrt{\frac{F}{\pi\delta}}.$$

For the steel wire

$$N = \frac{1}{\cdot 062 \times 30}\sqrt{\frac{10000 \times 981}{\pi \times 7 \cdot 9}},$$

For the brass wire

$$N = \frac{1}{\cdot 050 \times 25}\sqrt{\frac{F}{\pi \times 8 \cdot 4}}.$$

Equating and squaring we get

$$F = \left(\frac{\cdot 050 \times 25}{\cdot 062 \times 30}\right)^2 \left(\frac{10000 \times 981}{\pi \times 7 \cdot 9}\right) (\pi \times 8 \cdot 4) \text{ dynes.}$$

$$= \left(\frac{\cdot 050 \times 25}{\cdot 062 \times 30}\right)^2 \left(\frac{84}{7\cdot 9}\right) \text{ kilos.}$$
$$= 4\cdot 803 \text{ kilos.}$$

4. If four wires of the same length and material but of diameters in the ratio 1 : 2 : 3 : 4 are all stretched to half their breaking stress, compare the frequency of the notes they yield.

The areas of the four wires are in the ratio of the squares of the diameters, that is, in the ratio 1 : 4 : 9 : 16.

Since stress is equal to force per unit area and by hypothesis the stresses are equal, the stretching forces are in the ratio 1 : 4 : 9 : 16.

Let them be F, 4F, 9F, 16F, and let the diameters be d, $2d$, $3d$, $4d$.

Frequency of note given by the first string is

$$N_1 = \frac{1}{dl}\sqrt{\frac{F}{\pi\delta}}.$$

Frequency of note given by the second string is

$$N_2 = \frac{1}{2dl}\sqrt{\frac{4F}{\pi\delta}} = \frac{1}{dl}\sqrt{\frac{F}{\pi\delta}},$$

which is equal to N_1.

Similarly N_3, N_4 are equal to N_1, therefore the notes given by the four strings are of the same frequency.

Aliter. The formula

$$N = \frac{1}{dl}\sqrt{\frac{F}{\pi\delta}}$$

is equivalent to

$$N = \frac{1}{l\sqrt{\pi\delta}}\sqrt{\frac{F}{d^2}}.$$

We see that with our strings $\frac{F}{d^2}$ is a constant, therefore N is the same for each.

5. In an experiment with the sonometer, to prove that $N \propto \sqrt{F}$ two forks C and D were used and the forces required for the same length of wire were 20·5 and 16·5 pounds respectively. The pitches being 256 and 288, find the percentage error in the experiment.

6. In an experiment with the sonometer, to prove that with the same frequency $l \propto \sqrt{F}$, using the same wire, the following readings were taken for the lengths and stretching weights :

55·7,	50·2,	41·8 cms.,
16·5,	13·2,	9·0 kilos.

Find the percentage error in the experiment.

7. A steel wire a yard long and stretched with 5 lbs. weight gives a note of frequency 500. What force in lbs. weight would be needed to make two yards of the same wire vibrate twice as fast?

8. An iron telegraph wire when set in vibration by the wind was observed to give a note of frequency 100. Its span was measured and found to be 50 metres, and its diameter 1·5 mm. Assuming its density to be 8 gms. per c.c., find the stretching force in kilogrammes weight.

9. Two wires of the same material and thickness yield the same note when plucked. One of them is 36 inches long and carries 100 lbs. If the other is 45 inches long, find how many pounds it carries.

10. In an experiment with the sonometer on three different steel wires under the same stretching forces and yielding the same note, the lengths were found to be 40, 45, and 50 cms. respectively. The radius of the largest wire was ·45 mm.; find the radius of the others.

11. A wire 56 cms. long and carrying 7 lbs. has a mass of ·62 gm., and the note it yields is that of a certain fork. Find its frequency.

12. A wire a metre long weighing $2\frac{1}{2}$ gms. is stretched on a sonometer by a weight of 18 kilos., and when its length is adjusted to 80 cms. and a vibrating fork is applied the string throws off a rider placed at any point except two, besides the ends. Find the frequency of the fork.

13. Four violin strings of the same length and material but of diameters in the ratio 4 : 3 : 2 : 1 are to be stretched so that each gives a note a fifth above the preceding. Compare the forces necessary.

14. Find the pitch of a transversely vibrating piece of catgut $\frac{1}{2}$ metre long, 2 mm. diameter, and stretched by a force of 25 kilos. (density of catgut 1·7, $g = 981$).

15. A string, when stretched by 16 kilos., gives a certain note. By how many kilos. must it be stretched in order to give the major third (1) above, (2) below that tone?

16. Find the ratio of the notes given out by two wires of the same material, the second one having double the diameter, double the length, and being stretched by double the weight of the first.

CHAPTER IV.

LONGITUDINAL VIBRATIONS.

(1) In Air Columns.

The points of maximum change of pressure but of minimum motion are called **nodes**. Points of maximum motion but minimum change of pressure are called **antinodes**.

Resonant Column in Tube over liquid.

Bottom of column is a node, top is an antinode ; therefore lengths which resound are $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, etc., where λ is the wave-length of the note to which they resound.

The antinode is not quite at end of tube, but at a point ·8 of the radius beyond ; therefore the first length $= \frac{\lambda}{4} - \cdot 8r$, and so on for the others.

Difference between the lengths is, however, always $\frac{\lambda}{2}$.

Organ Pipes.

At the embouchure there is an antinode.

The top of the pipe is a node or antinode according as it is closed or open.

Keeping the length l fixed, we see that the lengths of stationary waves which can be formed in it are—

For open pipes	$\lambda = 2l$,	l,	$2l/3$, etc.
With frequencies	$2l/V$,	l/V,	$2l/3V$, etc.
And for closed pipes	$\lambda = 4l$,	$4l/3$,	$4l/5$, etc.
With frequencies	$4l/V$,	$4l/3V$,	$4l/5V$, etc.

(2) **Longitudinal Vibrations of Rods and Wires.**

Fixed points are nodes, free points are antinodes.

Hence for **rod clamped at the middle** and set in longitudinal vibration

$$\lambda = 2l,$$

and V, the velocity of sound in the rod $= n\lambda = 2nl$.

For wires fixed at their ends

$$\lambda = 2l.$$

(3) **Reflection in air at a hard wall.**

The surface of the wall is a node, and other nodes occur at consecutive distances of $\lambda/2$, measured normally from the wall to the source.

EXAMPLES IV.

1. A glass tube 60 cms. long and 2 cms. diameter open at each end is lowered vertically into water until the air column resounds to a G fork; on lowering still further another point of resonance is found. Find the lengths of the tube unimmersed in the two cases.

(G = 410 vibr. per sec., V = 34000 cms. per sec.).

$$\lambda \text{ for G fork} = \frac{34000}{410} = 80{\cdot}5 \text{ cms.},$$

$$\frac{\lambda}{4} = 20{\cdot}12, \qquad \frac{3\lambda}{4} = 60{\cdot}36.$$

For the first position $l + {\cdot}8 = 60{\cdot}36$,

$$\therefore \; l = 59{\cdot}6.$$

For the second position $l + {\cdot}8 = 20{\cdot}12$,

$$\therefore \; l = 19{\cdot}3,$$

therefore lengths unimmersed are 59·6 and 19·3 cms. respectively.

2. An open glass tube standing in water resounds at distances of 15 and 46·1 cms. to a C fork (freq. = 546), the temperature being 12°. Find the velocity of sound in air at 0°.

The difference between the two lengths is equal to $\frac{\lambda}{2}$,

$$\therefore \frac{\lambda}{2} = 31{\cdot}1.$$

$$\therefore \lambda = 62{\cdot}2,$$

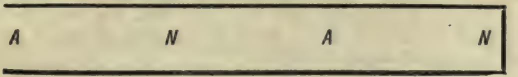
$$V_{12} = n\lambda = 546 \times 62{\cdot}2 = 33{,}960 \frac{\text{cms.}}{\text{sec.}}$$

To get V_0, subtract 60 cms. for every degree of temperature above zero, therefore

$$V_0 = 33960 - 12 \times 60 = 33{,}240 \frac{\text{cms.}}{\text{sec.}}$$

3. The air in a closed tube 34 cms. long is vibrating with two nodes and two antinodes and its temperature is 51°. What is the wave-length of the waves produced in the air outside the tube if the temperature of the air is 16°C. ?

Find the velocity of sound at 51°, then from the wave-length of the note in the tube find the frequency. The frequency is the same both in air and in the tube. Then from the velocity at 16° find the wave-length in air.



Velocity at 51°

$$V_{51} = 33240 + 51 \times 61 = 36351 \text{ cms.}$$

$$\frac{\lambda}{2} = \frac{2}{3} \times 34, \therefore \lambda = \frac{4}{3} \times 34 = \frac{136}{3} \text{ cms.}$$

$$V = n\lambda, \therefore n = \frac{36351 \times 3}{136}$$

$$V_{16} = 33240 + 16 \times 61 = 34210,$$

$$\therefore \lambda = \frac{34210 \times 136}{36351 \times 3} = 42{\cdot}6 \text{ cms.}$$

4. A glass rod 5 feet long clamped at its centre and rubbed longitudinally gives a note of frequency 1295. Calculate the velocity of sound in the glass.

The centre is a node the two ends are antinodes, hence for the fundamental vibration $\lambda = 2l$.

In our case $\lambda = 10$ ft., $\therefore v = 12950$ ft./sec.

5. A whistle is sounded near a wall and the nearest node to the wall is 3 cms. from it. Determine the frequency of the whistle.

Here $\frac{\lambda}{2} = 3$ cms., $\therefore \lambda = 6$ cms.

and $n = 33240/6 = 5540$.

6. What is the length in cms. of an open organ pipe, if its fundamental note has a frequency of 384 at 15°C. ?

7. The length of a tube, open at each end, containing air which most loudly resounds to a certain fork is 6 inches, the temperature being 15°. Find the frequency of the fork.

8. A brass rod 5 feet long clamped at its centre and caused to vibrate longitudinally yields the same note as an organ pipe 6 inches long at 12°. Calculate the velocity of sound in brass.

9. A tube closed at one end, filled with air and 12 inches long, resounds to a fork of frequency 300. What is the temperature of the air in the tube?

10. Calculate the length of a closed organ pipe which would yield a note of the same frequency as that of an open pipe 2 feet long.

11. In a Kundt's dust figure experiment the distance between the first and last of nine consecutive nodes was 72 cms., the length of the rod was 5 feet, the temperature was 12½°, and the frequency of the note emitted by the rod 1889. Find the velocity of sound in the rod and the velocity of sound in air at 0°.

12. If the fundamental note of an open organ pipe has a frequency of 100, what is the frequency of each of its first three overtones?

13. If the fundamental note of a closed organ pipe has a frequency of 50, what is the frequency of each of its first three overtones?

14. Give the lengths of the three shortest closed tubes, and of the three shortest open tubes which would resound to a fork of frequency 200.

15. A closed tube 15 cms. long resounds when full of oxygen to a given fork. Give the length of a closed tube full of hydrogen which will resound to the same fork.

16. A vertical tube 1 metre long and 4 cms. in diameter is gradually filled with water, while a tuning fork of 500 is sounded over its upper end. At what positions of the water surface will the tube resound, taking the correction for diameter into account, the velocity of sound being 33243 cms./sec.

MISCELLANEOUS EXAMPLES.

HEAT.

1. Calculate the temperatures Centigrade corresponding to 100° F., − 40° F., 0° F., 98° F.

2. What temperatures Fahrenheit correspond to + 30° C. and − 40° C.?

3. The temperature of a person was found to be 37·4° C. upon one occasion, and 101·2° F. upon another. Upon which occasion was he hottest, and by how much?

4. Describe some experiment to prove that iron and brass expand differently when heated. How is this fact made use of in a gridiron pendulum?

5. Calculate the extension, in inches, of a bar a mile long at 0° when heated from − 40° to + 35°, the coefficient of expansion of its material being ·000018.

6. A stay-rod of iron, 20 feet long, is subject to fluctuations of temperature from zero to 120° F. The coefficient of expansion of iron is ·0000123 per degree Centigrade. What is the maximum variation of length that must be allowed for?

7. Describe, carefully, some ways in which the force of contraction on cooling is usefully applied in the Arts. Describe also some inconveniences arising from expansion and contraction, and a mode of overcoming any one of them.

8. A railway girder, 87 metres long, whose coefficient of expansion is ·000018 per degree Centigrade, changes in temperature from − 10° C. to + 30° C. between winter and summer. Calculate its change of length.

9. If the linear coefficient of expansion of iron is ·0000123, find the increase in the capacity (in litres) of a cylindrical steam-engine boiler, which at the freezing-point is 7 metres in length and 3 in diameter, when heated from 15° to 150°.

10. A barometer with a glass scale reads 755 mm. at 18° C. Find the reading at 0° C. The apparent coefficient of expansion of mercury in glass is ·000155, and the coefficient of linear expansion of glass is ·0000089.

11. If the column of mercury in a barometer rises $1\frac{1}{3}$ centimetres when the instrument is taken out of ice and immersed in steam at atmospheric pressure, what is the absolute coefficient of cubical expansion of mercury? The pressure of the atmosphere being equal to 75 centimetres of mercury at 0°.

12. A specific-gravity flask, which when empty weighed 5 gr., weighed 150 gr. after it had been filled full of liquid up to a fixed mark when immersed in melting ice. It weighed only 147 gr. after it had been filled up to the same mark when immersed in steam. What is the apparent coefficient of expansion of the liquid?

13. An open flask is heated from 0° to 100° C. What fraction of the mass of the air is expelled? You may neglect the change in volume of the flask.

14. If 3,000 cubic inches of air at 0° C. expand by 11 cubic inches for each degree rise of temperature, find the volume at 100° of a quantity of air which at 50° measures 100 cubic inches, the pressure being supposed to undergo no change.

15. The volume of a mass of gas at a pressure of half an atmosphere and temperature 15° C. is 150 c.c. Find the volume when the temperature is 303° C. and the pressure one atmosphere.

16. A vessel weighing 250 grammes is made of a metal with specific heat 0·1. The vessel is at 15° C. initially and empty. 500 grammes of water at 100° are poured in. Find the common temperature.

17. Describe an experiment by which you would show that ice contracts when it melts, and that the resultant water goes on contracting if it be warmed.

18. To a pound of ice at 0° are communicated 100 units of heat (pound-degrees Centigrade). What change of temperature does the ice undergo, and in what way is its volume altered?

19. If dry ice, at its melting-point, be thrown into 20 lbs. of water at 60° C. until the whole weighs 23 lbs., what fall of temperature will have been thereby produced? How much mechanical work must be done on the water in order to restore its original temperature?

20. The latent heat of steam is 536; that of water, 80. What weight of steam at 100° C. would be required to melt a kilogramme of ice?

21. Alcohol boils at 78° C.; its latent heat of evaporation is 202; and its mean specific heat, when liquid, is ·65. Calculate the least quantity of water at 10° C. needed to condense 100 gr. of alcohol from vapour at 78° C. into liquid at 15° C.

22. The latent heat of alcohol vapour being 202, and of water vapour 537, thermal units per gramme at atmospheric pressure, calculate what is the mass of steam at 100° C. that must be passed through a worm in an alcohol still in order to evaporate a kilogramme of alcohol, if it was already at its boiling-point, 77° C.

23. If the latent heat of vaporisation of water at 100° C. be 537, how much heat per hour would be given out by a stack of pipes

in which 0·12 kilogrammes of steam, at atmospheric pressure, was condensed per minute?

24. What are clouds? Explain two modes in which you suppose they may be formed in nature. How would you form an artificial cloud?

25. Enough water is placed in a closed vessel to saturate the air it contains at 15°, and the pressure of the dry air in the vessel at 0° is 760 mm. Find the pressure at 25°. The saturating pressure of vapour at 15° is 16·1 mm.

26. A clear piece of rock-salt, a bright piece of tin-plate, and a piece of black wood are exposed to the radiation of a common fire. State what happens in each case, and what is the final state of each as regards temperature.

27. If sunshine falling on ice can melt a layer one-eighth of an inch thick in an hour, how much water could it evaporate in the same time from a lake one-tenth of a square mile in area? The latent heat of ice melting is 80, and of water evaporating 600, and the density of ice is ·920.

LIGHT.

1. A bar, 18 centimetres from a screen, casts two shadows close together, of equal shades, when the two lights are respectively 20 centimetres and 30 centimetres distant *from the bar*. Calculate the ratio of the intensities of the two lights.

2. A circular uniform source of light, 2 inches in diameter, is placed at a distance of 10 feet from a sphere 2 inches in diameter. Calculate, approximately, the diameters of the umbra and the penumbra cast on a screen 5 feet beyond the sphere.

3. When the sun shines through a very small hole, it can make, on a screen 560 centimetres distant, a circular spot 5 centimetres in diameter. Using this datum, calculate the width of the penumbra of the shadow of an object, in full sunshine, upon a screen 10 metres distant from the object.

4. Show, by carefully drawn diagrams, that a man walking up to a mirror will see in the mirror more and more of the landscape behind him as he approaches. Is this true also of his own figure?

5. A small source of light is submerged in water. Draw the paths of a number of rays issuing from it and striking the surface of the water. Indicate what becomes of each of them, making a construction for the critical angle. The index of refraction of water may be taken as $\frac{4}{3}$.

6. Light falls at a given angle on a plane refracting surface, for which the refractive index is $\frac{5}{4}$. Show, by a geometrical construction, drawn, as well as you can, to scale, how to find the direction of the refracted ray.

7. Explain clearly how to determine the real position of an object seen straight down under water, from observation of its apparent position. Does it appear at the same place when viewed perpendicularly to the surface, and when viewed in an oblique direction? Give reasons for your answer, with diagrams.

8. Describe (with a diagram) how to measure experimentally the distance between an object and its real image in a concave mirror.

9. A concave mirror of 2 feet focal length is placed 1 foot from an object. Find the change in the position of the image produced by moving the object 1 inch nearer the mirror.

10. A bright object, 4 inches high, is placed on the principal axis of a concave spherical mirror, at a distance of 15 inches from the mirror. Determine the position and size of its image, the focal length of the mirror being 6 inches.

11. An object, 2 centimetres high, is placed 1 metre away from a spherical concave mirror of 23 centimetres radius of curvature. Calculate the height of the image. Will it be real or virtual?

12. Draw a diagram explaining the formation of an image of a given object by a concave lens. Can such an image be made larger than the object? Give reasons for your answer.

13. Draw carefully, and as nearly as you can to scale, the course of a pencil of light, starting from an object 3 inches from a convex lens whose focal length is 2 inches. Find the relation between the size of the object and that of the image.

14. Find the focal length of the lens needed to form an image seven times as large as the object, on a screen 24 feet from the object. Will it require a convex or a concave lens? Give reasons for your answer.

15. A lens of focal length of 1 foot is placed at distances (*a*) of 3 feet, and (*b*) of 6 inches, from an object 2 inches long. Draw figures showing the paths of rays of light from each extremity of the object, and find the position and size of the images formed in the two cases respectively.

16. A concave lens of 10 centimetres focal length is placed 50 centimetres from an object 2 centimetres high. Calculate the size of the image, and draw a diagram of the rays by which it is seen. Is it real or virtual? Is it erect or inverted?

17. A person looks at an object through a concave lens of 1 foot focal length, the object being 5 feet beyond the lens. Draw a figure showing the paths of the rays by which he sees the image formed, and determine its position.

18. A figure on a lantern slide is 1 inch long. The screen is 13 feet from the slide, and a single lens is to be used to throw the image on the screen. Where must the lens be placed, and what must be its focal length, in order that the image of the figure on the screen may be 1 foot long?

19. A short-sighted person, who can see most distinctly at a distance of 6 inches from his eye, wishes to see an object 5 feet off. What sort of a lens should he use, and what must be its focal length? Illustrate your answer with a figure.

20. Draw the course of a ray of light through a right-angled isosceles glass prism, the ray being incident at right angles to the face opposite the right angle.

21. Draw, as well as you can to scale, the path of a ray of simple light passing symmetrically through a prism whose refractive index is $\frac{3}{2}$, and whose refracting angle is 45°.

22. A bright object is observed through a prism whose refracting edge is horizontal and uppermost. Will the top of the object, as seen through the prism, appear to be red or blue? Give reasons for your answer.

23. A spectrum of the light from a slit is formed by a prism in a position of minimum deviation, the distances between the slit and prism, and prism and screen, being each 4 feet. A convex lens of 2 feet focal length is then introduced close up to the prism. How is the spectrum altered? Draw figures to illustrate the two cases, showing in each the path of the light.

24. Explain, with a diagram, how to place a slit, a screen, a lens, and a prism, so as to form a pure spectrum of sunlight, and, if possible, to exhibit the Fraunhofer lines.

25. When white light passes through a red liquid and goes out red, has it acquired its colour from the liquid? If not, explain precisely what has happened, and show how to prove your assertions experimentally.

26. Describe how the velocity of light can be obtained by observations made on the positions of fixed stars, with respect to the zenith, at different seasons of the year.

27. If light takes 8 minutes to come from the sun, which is 92,000,000 miles off, how long will it take to go 3,000 miles and back?

SOUND.

1. Find the wave-length of a note of 400 vibrations per second in steel in which the velocity of propagation of sound is 500000 cms. per second.

2. What is the name given to the interval which is the difference between an octave and a fourth?

3. How do you account for the fact that when water-waves travel obliquely towards a gradually sloping shore they become more and more parallel to the shore?

4. Two sounds each consisting of a fundamental tone with its first two harmonics reach the ear together. How many beats per second are due to the fundamental tones, how many to the first harmonics and how many to the second harmonics?

5. A string whose mass per cm. is ·005 gm. is stretched by a force of 8 million dynes; find the pulse-velocity.

6. The density of platinum is 22, and that of iron 7·8. What must be the ratio of the lengths of two wires, one of platinum and the other of iron, both of the same section, that they may vibrate in unison when stretched with equal forces?

7. At what temperature is the velocity of sound in nitrogen (atomic weight 14) equal to its velocity in oxygen at 10° (atomic weight 16)?

8. The velocity of sound in air is 34000 cms. per second when the temperature is 13° C. and the pressure that due to 78 cms. of mercury. What will be the velocity when the thermometer is at 22° and the barometer at 72 cms.?

9. Find the intervals between successive pairs of a fundamental note and its harmonics. When will the beats between the different harmonics begin to get disagreeable?

10. Suppose two forks of frequencies 256 and 272 are sounding together. Find the frequency of the beats and of the first difference and first combination tones.

11. A resonance tube responds to a fork of frequency 500 when the length of the column is 16·2 cms. The velocity of sound being 34000 cms. per second, find the end correction in terms of the radius.

12. At what temperature is the velocity of sound in air double the velocity at 0° C.

13. Calculate the velocity of sound in water vapour at 20° C. Given that $\gamma = 1{\cdot}3$, and that the density of hydrogren under normal conditions is ·0000896 gms. per c.c.

14. A train travels at 60 miles an hour. Find the frequency of the notes heard (1) by an observer in the train before and after passing a fixed whistle making 500 vibrations a second; (2) by a stationary observer before and after the engine passes him, the engine whistle making 500 vibrations a second. Show that the ratio of the frequencies of the two notes is the same in case (1) as in case (2). Velocity of sound = 1100 feet per second.

15. If two strings whose frequencies are 301 and 600 are sounded together beats are heard. Why? and what is the frequency of the notes causing them?

16. Why does sound spread further on a clear night than on a sunny day? Why have intervening obstacles much less effect in the former case?

17. Prove that when sound waves spread through free air in all directions, the intensities at different points are inversely as the squares of the distances of those points from the source.

18. Compare the pitches of an organ on two days when the temperatures were 14° F. and 86° F.

19. How long will a sound wave take to travel down a shaft of a mine 3000 feet deep if the temperature at the top is 0° C., and at the bottom 30° C. (velocity at 0° = at 1090, and increases 2 feet for every rise in temperature of 1° C.)?

20. When a rod of rectangular cross section vibrates transversely the frequency of the note is proportional to its depth. If the sides of the rectangle are 1·2 and 1·8 cms. what is the interval between the notes emitted by the bar in the flat and edge positions?

ANSWERS.

HEAT.

EXAMPLES I.

4.—(1) −94° F. ; −56° R.
(2) $24\cdot\dot{4}$° C. ; 19·5° R.
(3) −30° C. ; −22° F.
(4) 32° F. ; 0° R.
(5) 10° C. ; 8° R.
(6) 20° C. ; 16° R.
(7) 80° C. ; 176° F.
(8) 197·6° F. ; 73·6° R.
(9) −10° C. ; −8° R.
(10) 62·5° C. ; 144·5° F.

EXAMPLES II.

1. 1·000089.
2. 1.
3. 1·0007 ; 999961.
4. ·999975 ; 1·0013.
5. 1·00012 ; 1·00032.
6. 100·084 ; 99·984.

EXAMPLES III.

8. ·03 cm.
9. $72\cdot\dot{2}$ cms.
11. 30·77° C.
12. $\cdot 00001\dot{3}$.
13. ·000018 ; 2·91.
15. 124·94° C.
16. 100·008 cms.
17. ·0012 V.
18. ·000025 ;
99·85 ;
99·75.
19. ·000007224 (nearly).
20. V (1 + 30 c + 100 c').
$$\frac{(30c + 100\,c')}{130\,(1 + 30\,c)}.$$

EXAMPLES IV.

4. $\dfrac{d_{100}}{d_{-100}} = \cdot 9832.$
5. $(1 + c\,t)$; $(1 - c'T)$.
6. 10·1476 grams.
7. −·00003225.
8. $\dfrac{\delta - \delta'}{\delta\,(t' - t)}$ $[t' > t]$.
9. 193·869 grams.
10. 386·25° C. (nearly). [Work by formula 1, Art. 24.]

EXAMPLES V.

11. ·000017.
12. – ·000032496 (nearly).
·000433277.
·00010908.
13. ·000182.
14. 9·04 grams [c_a = ·000158].
15. ·000185.
16. 100° C.
17. ·000029.
18. 80 c.c.;
81·3347;
·000174 (nearly).
19. 764·9462 cm. [c_r = ·0001815].
20. ·00002783.
21. $\frac{1 + \delta}{1 + \Delta}$.

EXAMPLES VI.

8. 0·003821.
9. 0·003813.
10. 154·7.
11. 663·77.
12. (1) $P, T', \frac{T' V}{T}$.
(2) $\frac{T' P}{T}, T, V$.
13. $\frac{1}{c_r} = 273{\cdot}68$.
14. 19° C.
15. 76·23 cm.
16. 22·79 c.c.
17. ·0841 gram.
18. 1·00428 (nearly).
19. 313° C.
20. 273° C.
21. 93·75 atmospheres.
22. –459·4° F.

EXAMINATION QUESTIONS (page 26).

Elementary.

2. 534·16 c.c.
3. 450·531 cub. in.
12. 64·1° C.
15. ·185 cub. ft. (nearly).
16. 1·54 c.c.
17. 345·2 cub. in.; 577·4 cub. in.

Advanced.

19. 1·0543 kilogrammes.
23. 98·6° C. (nearly).
24. 29·457 in.
28. ·00006.
31. ·3432 gram increase.
33. 195:112 (assuming g to be constant).

EXAMPLES VII.

8. ·0311.
9. 25·89°; 9·12 grams.
10. 13·1579 grams.
11. ·458.
12. ·0903 (nearly).
13. ·6153.
14. 13·21°.
15. 745·3°.
16. 77·29° (nearly).
17. 1·005; 1·027; 1·016.
18. $\frac{M(s't' + s''t'') + mst}{M(s' + s'') + ms}$.
19. 52·33° (nearly).
20. 3·41.

Examples VIII.

4. 95·58 pound-degrees.
5. – 3·6° C.
6. ·9047.
7. 79·56$\dot{1}$.
8. ·1108.
9. ·0334.
10. ·9167.

Examples IX.

4. 1117·04 c.c
5. ·0242 c.c.
6. 4·098.
7. 17·391 mm. ; 382·609 mm.
9. k = ·211$\dot{6}$ (taking the hour as unit of time).
   ·8224 grams per hour.
10. 674764·76 dynes per sq. cm.
11. 966·$\dot{6}$; 144.
13. 72·75 grams.
14. 564·98.
15. 65·3° C.
16. 2900 grams (*q.p.*).
17. 90·34° C.
18. 2024 gram-degrees.
19. 35·11° C.
20. 1·9781 grams.

Examination Questions (page 43).

Elementary.

2. 1:715·25.
5. ·092.
6. 320° C.
8. ·109$\dot{5}$ (nearly).
10. 5·66 lbs.
13. 12·5° (nearly).
15. 44·$\dot{4}$ gram-degrees.
16. 495·3.
17. 25:2 ; 1·85$\dot{1}$° C.
18. ·0329.
19. 31·85°.
20. ·932.
21. 79·705.
23. 48·923°.
24. 80.
26. 47·5 gram-degrees ; 59·375 grams.

Examples X.

4. 1·205 grams.
5. –5·2° C.
6. 10·1° C. ; 77·1 per cent.
7. p = 72·76 per cent.
8. ·0166 gram.
9. 46 per cent. ; ·0079 gram.
10. 12·366 grams.

Examples XI.

5. ·0384.
6. 12,000,000 gram-degrees.
7. 125·28 gram-degrees.
8. 4,312,500 gram-degrees.
9. ·0001$\dot{3}$.
10. 1800.
11. ·192 (nearly).
12. ·0672.

Examination Questions (page 54).

Advanced.

12. 56·3 per cent.

15. 90·7 c.c.

Examples XII.

5. ·084.

6. 96·65.

7. 5·7 per cent.

8. 29266·6 *centimetre-grams*, or 29272·75 *centimetre-grams.*

10. ·1683 ; ·2373.

LIGHT.

Examples I.

7. 100 cm. ; 20 cm.

8. Diameter of umbra = 6·71 cm.
Diameter of penumbra = 8·904 cm.

9. ·00894 sq. cm. (nearly) ; $A'B' = 13{\cdot}\dot{3}$ cm.

10. 3 : 4.

11. $I_D : I_B : I_C :: 3\sqrt{3} : 8 : 8$.

12. $a^2 : b^2$.

13. $(115)^2 : (201)^2$.

14. 80 cm. from less intense light.

15. (*a*) Screen between the lamps, 2⅔ ft. from 16-power lamp.
(*b*) Screen outside lamps, 24 ft. beyond 16-power lamp.

Examples II.

4. 12 in., 24 in., 36 in. ; 12 in., 12 in.

5. 60°.

6. 3 ft.

7. 100°.

8. 7 times.

Examples III.

7. (1) Distance > 12 ; (2) distance < 12.

8. 13·5.

9. $\frac{I}{O} = \frac{f}{u-f} = \frac{f}{3f-f} = \frac{1}{2}$.

10. 12.

11. ·025 in. ; 9 in. behind.

12. $r = 5\frac{1}{3}$ ft.; mirror $13\frac{1}{3}$ ft. from wall.
13. $3\frac{3}{7}$ in. behind; $\frac{3}{7}$.
14. 2 ft. real image, 1 ft. virtual.
15. Real image, size $\frac{1}{5}$ object.
17. $\frac{1}{2}$, virtual, $\frac{f}{2}$ behind.
18. 7·5, 30 cm. from mirror.
20. $2\sqrt{3}$ from plane mirror.

Examination Questions (page 76).

9. Real, and one-third as large as the object; 1 ft. from the mirror; inverted.

16. 6 in.

19. 1 ft. from the mirror; inverted; three times as large; at the centre of curvature.

21. Object 3 in. in front of mirror; image $1\frac{1}{2}$ in. behind the mirror.

26. Between candle and gas flame, 2 ft. from former and 6 ft. from latter; or, on the line passing through the lights, 4 ft. from the candle and 12 ft. from the gas flame.

Examples IV.

5. $1\frac{7}{8}$ nearer.
7. $\frac{9}{7}$.
8. 1·68.
10. $\frac{2\sqrt{3}}{3}$.
11. 4·38.
12. 225563909·8.
13. 1·2.
15. $2\frac{2}{3}$.

Examples V.

9. ·625°.
10. $\sqrt{2}$.
12. 1·5.
13. $-83\frac{1}{3}$.
15. $3 \pm \sqrt{3}$.
16. Real; 5 in.
17. If image is real, $f = 8$ cm.
 „ „ „ virtual, $f = 13\frac{1}{3}$ cm.
18 Virtual, $\frac{10}{7}$ object; $4\frac{2}{7}$.
19. 9 in.
20. 15 in.
21. -12.
22. 24.
23. 4.
25. 10 in. in front of plane mirror.
26. $10\sqrt{2}$ from mirror.
27. 2·25.

EXAMINATION QUESTIONS (page 89).

3. 25 : 1.

5. 5 in.

6. 5 in. from the bright point.

8. The images formed are 6 in. and $8\frac{1}{4}$ in. below the surface of the water.

11. $\frac{3}{4}$.

17. 18 in. (To obtain a *real* image with a convex lens.)

20. $\frac{9}{8}$.

23. First image $8\frac{4}{7}$ in. from first lens ; diameter $\frac{3}{7}$ in. Second image $15\frac{45}{61}$ in. from second lens ; diameter $\frac{15}{61}$ in.

26. 4 in.

28. See Ex. V. 5.

31. 1·48.

33. 2 ft. from lens on the same side as the object ; 6 in. high.

EXAMINATION QUESTIONS (page 93).

8. 11 in.

9. On the same side as the object and 12 in. from the centre.

11. Convex ; 75 cm.

14. $2\frac{1}{4}$ in. or $4\frac{1}{2}$ in.

SOUND.

EXAMPLES I.

5. 483,000 cms. per second.
6. 3,360 feet.
7. 500,000 cms. per sec.
8. 85 metres.
9. 5 seconds.
10. 24·9° C.
11. 1100 feet per second.
12. 2 secs. after the sound was produced. 1100 feet.
13. 1131 feet per second. 21° C.
14. $1{\cdot}2 \times 10^{12}$ dynes per sq. cm.
15. 565 feet.
16. 17·5° C.

EXAMPLES II.

6. 22 miles an hour.
7. 5.
8. 2 feet.
9. 110·8 cms., 131·1 cms.
10. 900 feet per second.
11. 36·3.
12. 31.
13. 461·54.
14. 450.
15. 256. 44 feet.
16. The C above G.

EXAMPLES III.

5. $1^{\circ}/_{\circ}$.
6. $1^{\circ}/_{\circ}$ nearly.
7. 80 lbs.
8. 144100 kilos.
9. $156\frac{1}{4}$ lbs.
10. ·40 and ·36 mms.
11. 149·8.
12. 498·2.
13. 1024, 1296, 1296, 729.
14. 214.
15. (1) 25, (2) 10·24 kilos.
16. $4 : \sqrt{2}$.

EXAMPLES IV.

6. 44·45.
7. 1120.
8. 11140 feet per second.
9. 55° C.
10. 1 foot.
11. 578,000 and 33,240 cms. per second.
12. 200, 300, 400.
13. 150, 250, 350.
14. Closed tubes—41·55, 121·65, 207·75 cms. Open tubes—83·1, 166·2, 249·3 cms.
15. 60 cms.
16. 81·5, 48·3, 15·0 cms. from the top.

MISCELLANEOUS EXAMPLES.

Heat.

1. $37\frac{7}{9}$° C. ; -40° C. ; $-17\frac{7}{9}$° C. ; $36\frac{2}{3}$° C.
2. 86° F. ; -40° F.
3. Second ; 1·04° C. or 1·88° F.
5. 85·536 in.
6. ·1968 in.
8. 6·264 cm.
9. 246·58425 litres.
10. 752·652 mm.
11. ·000178 nearly.
12. ·000211.
13. $\frac{100}{373}$.
14. $115\frac{37}{71}$ cub. in.
15. 150 c.c.
16. $95\frac{20}{21}$° C.
18. 20° C.
19. 18·26° C. ; 583,800 ft.-lbs.
20. 126 gms.
21. 4,859 gms.
22. 360·78 gms.
23. 3,866,400 water-gram-degrees.
25. 846·2 mm.
27. 222,500 lbs. = 100 tons nearly.

Light.

1. 361 : 576.
2. 2 in. ; 4 in.
3. $8\frac{13}{14}$ cm.
9. $\frac{4}{13}$ in.
10. Between object and mirror, 10 in. from mirror ; $2\frac{2}{3}$ in.
11. $\frac{46}{177}$ cm. ; real.
13. Image = twice object.
14. Convex.
15. (*a*) $1\frac{1}{2}$ ft. on the other side of the lens; image $\frac{1}{2}$ size of object.
    (*b*) 1 ft. from the lens on the same side; image twice size of the object.
16. $\frac{1}{3}$ cm. ; virtual ; erect.
17. 10 in. from lens.
18. 1 ft. from slide ; convex ; $-\frac{12}{13}$ ft.
19. Concave ; $6\frac{2}{3}$ in.
27. ·0313 sec.

Sound.

1. 1250 cms.
2. A fifth.
3. Refraction ; velocity decreases as depth decreases.
4. 2, 4, 6.
5. 40,000 cms. per second.
6. Length of iron wire = 1·68 times the length of the platinum wire.
7. – 25·4° C.
8. 34,547 cms. per second.
9. Octave, Fifth, Fourth, Minor Third, Major Third. After the sixth.
10. 16, 16, 528.
11. ·8 times the radius.
12. 819° C.
13. 41680 cms. per second.
14. 540, 460. 543·5, 462·9. Ratio 27/23.
15. 2 beats between 301 and 299 and also 2 beats between 600 and 602.
16. Wave fronts bend down towards the earth.
18. 115/107.
19. 2·68 seconds as a first approximation.
20. A fifth.

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